Module 9

Mathematics 6

Probability

Learning Technologies Branch

Alberta LEARNING

Mathematics 6

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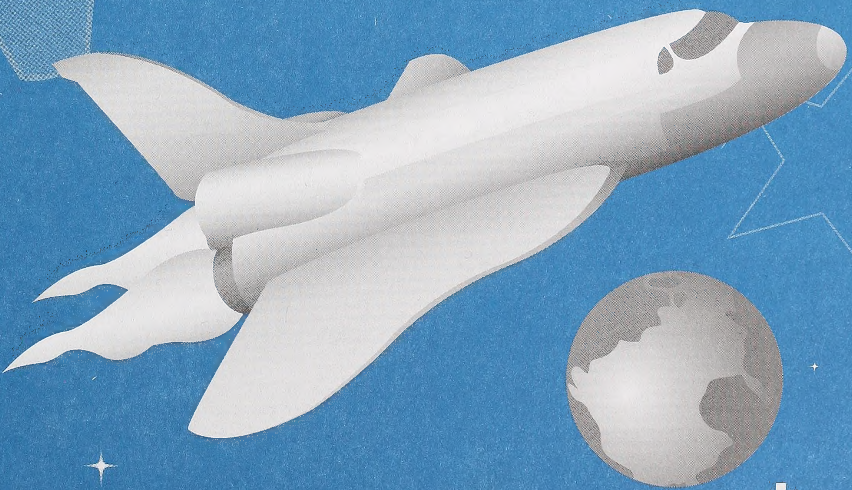
Probability



Learning Technologies Branch

Alberta LEARNING

Mathematics 6
Module 9: Probability
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome to Mathematics 6

Mathematics 6 contains nine modules.
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
Estimating and Representing Number

Module 2
Number Operations

Module 3
Patterns

Module 4
Fractions, Ratio, and Percent

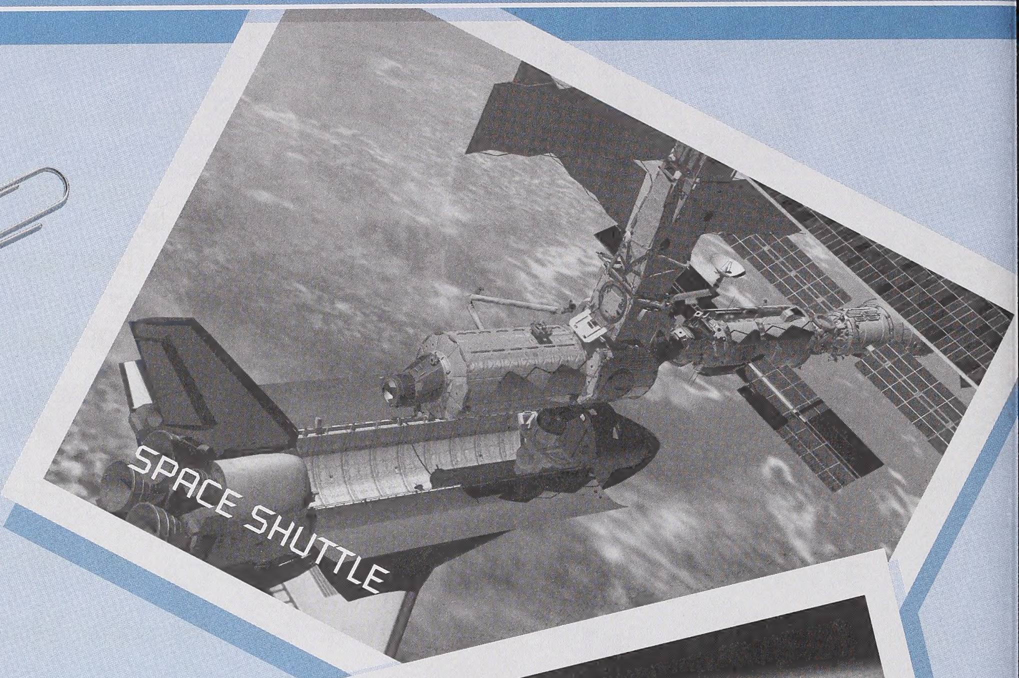
Module 5
Measurement

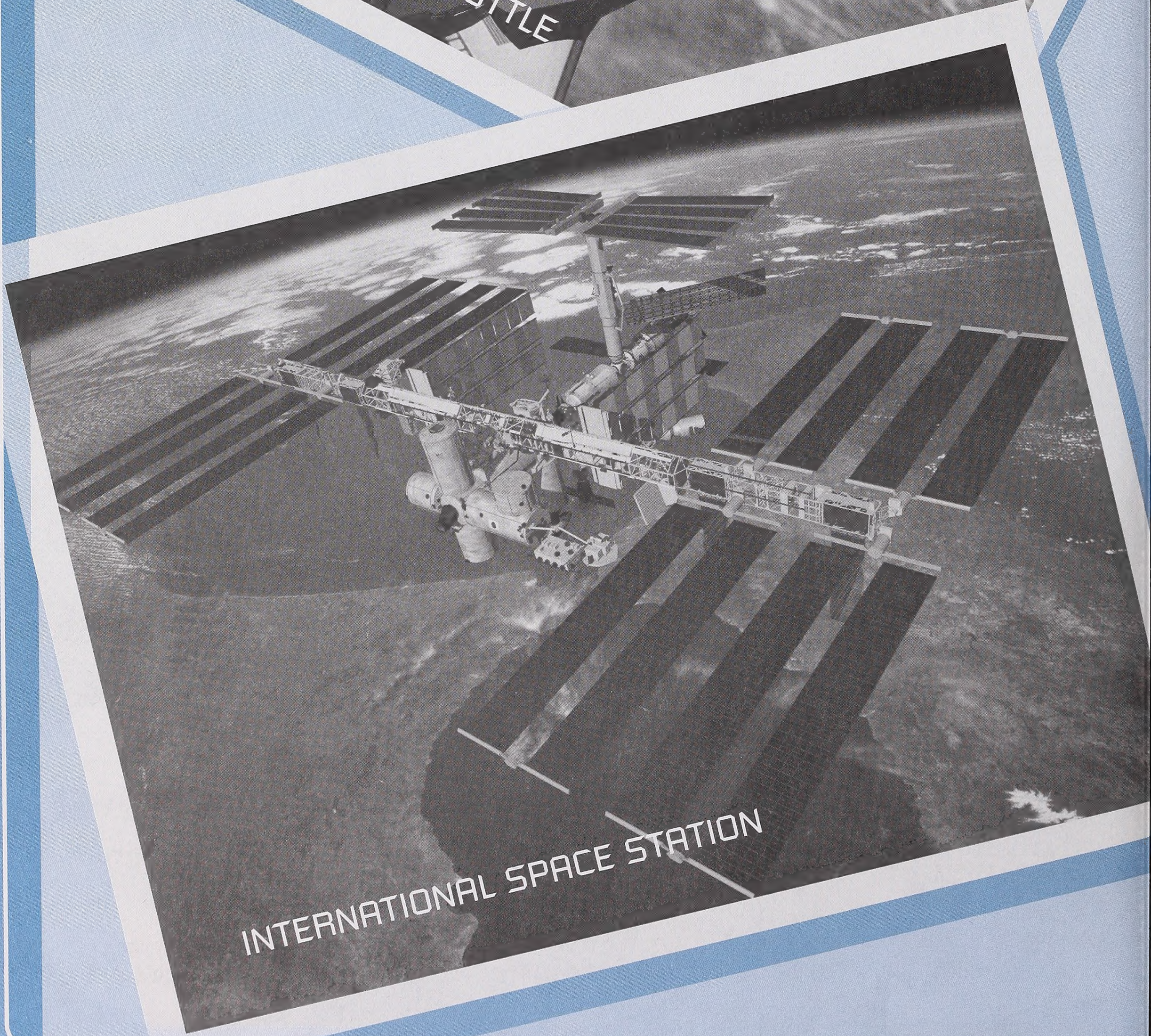
Module 6
Angles, Shapes, and Objects

Module 7
Transformations

Module 8
Data Analysis

Module 9
Probability


SPACE SHUTTLE


INTERNATIONAL SPACE STATION

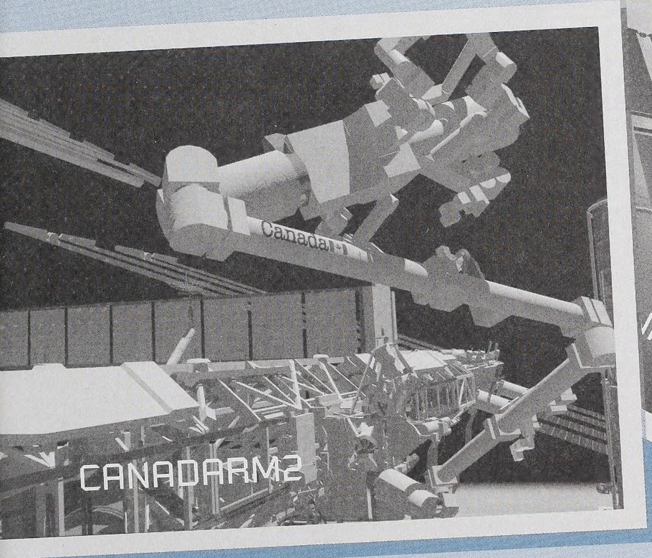
Adventures in Outer Space

Matthew: Wow, what a wonderful experience it was meeting Colonel Chris Hadfield at the Odyssium! He gave a presentation here in Edmonton on July 9, 2001, and talked about his adventures in space, including his mission aboard the Space Shuttle *Endeavor* to attach Canadarm2 to the International Space Station.

It's too bad you missed it, Kylee. You were away visiting your grandmother in Slave Lake.

Kylee: My trip was great, but I sure wish I could have heard Colonel Hadfield talk about being the first Canadian to walk in space. But I've got great news for you, Matthew! Commander Claire from the International Space Station is coming to town, and you and I will be spending some time with her.

I can't wait to hear about her adventures in space!



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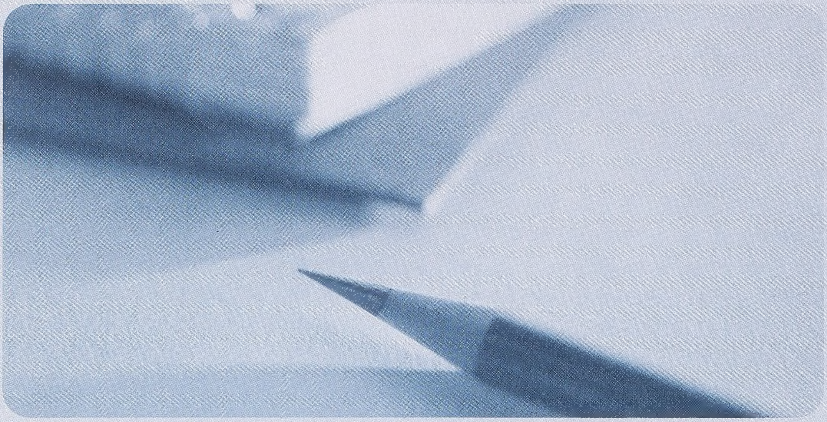
Commander Claire

Course Features



Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.
- Each lesson has several activities. An activity in each lesson is related to the Adventures in Outer Space theme.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module has special exercises that focus on certain mathematical skills. The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some “funky features” of the calculator that can be used to explore and practise important number ideas. Just the Facts gives you the opportunity to practise your basic number facts by doing a timed drill with your home instructor. The Mental Math exercise introduces an estimation skill or mental-computation strategy that you can use to sharpen your mental math skills.
- Each module references the Mathematics 6 Companion CD that includes additional material for review and mastery.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 6 textbook, *Quest 2000: Exploring Mathematics*, *Grade 6*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book*, *Grade 6*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the Mathematics 6 Companion CD.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.

Use your calculator.

Use the Internet.

Refer to the textbook or the Practice and Homework Book.

Use the Mathematics 6 Companion CD.

Assessment and Feedback

The Mathematics 6 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.

If you are having difficulty with an activity, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.

Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.

After your teacher marks an assignment, be sure to review your teacher's comments and correct any errors you made.

There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

Module Overview

Have you considered firefighting as a career? The men and women of your local fire department often put themselves at risk to protect lives and property in your community. They are also advocates of safety around homes and businesses to reduce the probability that their services will be required. However, life is not risk-free. Homeowners and businesses purchase insurance in case the worst happens. Insurance is based on the principle that risk is shared by all who purchase policies.

In this module you will explore chance and probability and how probability is calculated.

Lesson 1
Making Predictions

Lesson 2
Outcomes and Chance

Lesson 3
Games and Fairness

Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 9A

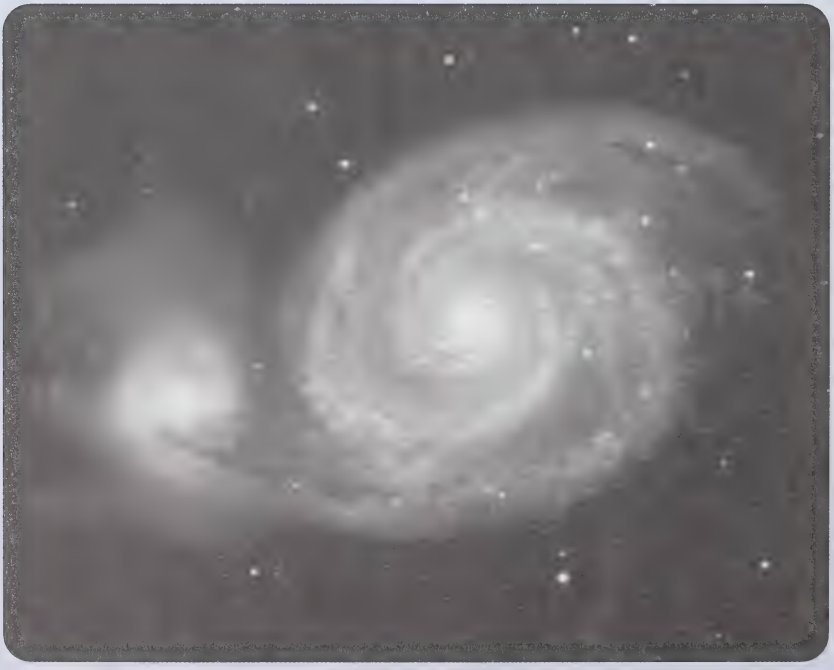
Lesson 1 Assignment	30 marks
Lesson 2 Assignment	30 marks

Assignment Booklet 9B

Lesson 3 Assignment	30 marks
Numbers in the News	10 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 9A to your teacher before you begin Lesson 3. You will submit Assignment Booklet 9B to your teacher at the end of this module.



Numbers in the News

Read through the following list before you begin Module 9. Begin by collecting samples of the ideas you already understand; others you may collect as you learn about them in the module. The samples you collect will depend on the newspapers or magazines you use.

Scavenger Hunt

Cut out articles or advertisements from newspapers or magazines that show predictions and probability being used in different situations. Here are some suggestions of things to look for:

- articles about probability
- predictions being made
- headings that use probability words, such as *predict*, *possible*, *impossible*, *certain*, *likely*, *unlikely*
- probability used in promotions

You will find further instructions for completing and submitting your project in Assignment Booklet 9B.

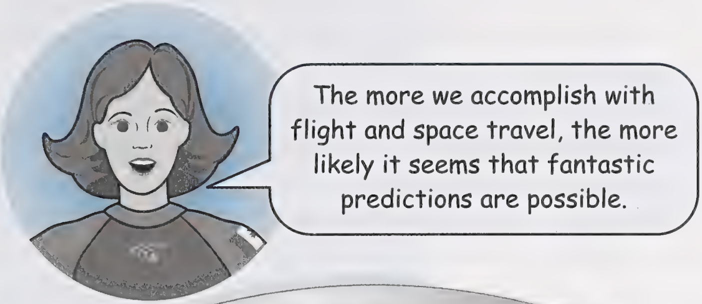
Making Predictions

When you plan outdoor activities in the summer, do you check the weather forecast to see if it is likely to rain? The chances of rain are expressed as probabilities. Meteorologists express these probabilities as percents. On many television forecasts, the term “probability of precipitation” occurs so often that it is abbreviated as POP. The weather forecast is just one area of everyday life that uses probability and the language of probability.

In this lesson you will explore how probability vocabulary is used when making everyday predictions. You will see that events do not always occur in the way they are predicted. You will use percents and equivalent fractions to express probabilities.

Activity 1

Today you will explore the vocabulary of **probability**.



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"I confess that in 1901, I said to my brother Orville that man would not fly for fifty years . . . Ever since, I have distrusted myself and avoided all predictions."

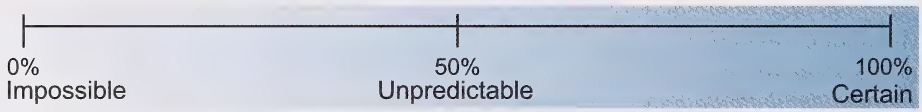
– Wilbur Wright
in a speech to the Aero Club of France, 1908

Ever since humans first looked to the sky in wonderment, the thirst for knowledge about the universe and anticipation of space travel has prompted numerous predictions. These predictions often describe personal judgments made by knowledgeable people about how likely particular events are to occur. Many of these predictions have never happened, many have been astonishingly accurate, and some have even been surpassed.

The success or failure of some predictions, such as human space travel to distant planets, may not be known for centuries. On the other hand, the success of predicting the winner of a hockey game may be known almost immediately.

Probability is often expressed on a scale from 0% to 100%.

If it is **impossible** for an event to happen, the probability of it happening is 0%. If an event is **certain** to happen, the probability of it happening is 100%.



There are many probability terms that are often used in making predictions. Here is a list of some basic probability terms.

- absolutely
- always
- can't happen
- certain
- could happen
- definitely
- for sure
- impossible
- improbable
- likely
- maybe
- must happen
- never
- perhaps
- possible
- probable
- should happen
- unlikely
- usually
- will happen

1. What terms in the list are similar in meaning to the following terms? Explain your answers.

a. impossible
b. certain

2. What terms in the list are appropriate in suggesting the following chances of something happening?

   a. between 0% and 50%
   b. between 50% and 100%
   c. anything except 0% and 100%

Check your answers on page 70 in the Appendix.

The following time line lists predictions made by famous people about the possibility of flight. It also lists actual events that happened. Use this time line to answer questions 3 to 6.

- **1895:** Lord Kelvin, Royal Society President, said, “Heavier-than-air flying machines are impossible.”

- **1902:** Simon Newcomb said, “Flight by machines heavier than air is unpractical and insignificant, if not utterly impossible.”

- **December 17, 1903:** Orville Wright made the first heavier-than-air, machine-powered flight in the world to carry a man.

- **1908:** Orville Wright predicted, “No flying machine will ever fly from New York to Paris . . . [because] no known motor can run at the requisite speed for four days without stopping.”

- **1924:** In the July edition of *Popular Science*, Edward Rickenbacker wrote, “Within the next few decades, autos will have folding wings that can be spread when on a straight stretch of road so that the machine can take to the air.”

- **1927:** Edward Rickenbacker made the first non-stop solo flight from New York To Paris.

- **1943:** Aviation publicist Harry Bruno predicted, “Automobiles will start to decline almost as soon as the last shot is fired in World War II. The name of Igor Sikorsky will be as well-known as Henry Ford’s, for his helicopter will all but replace the horseless carriage as the new means of popular transportation.”

3. Use the list of probability terms to help you write phrases that describe the following events.

   a. what Lord Kelvin thought in 1895
   b. what Simon Newcomb thought in 1902

4. a. When were the predictions from question 3 proven incorrect?

   b. What does that tell you about the science of flight in the early twentieth century?

5. a. Which of the terms from the list describe what Orville Wright thought in 1908?

   b. Explain why Orville's prediction was likely to be accepted as true.

   c. What actually happened regarding Orville's prediction?

6. a. Which of the probability words describe both Edward Rickenbacker's 1908 prediction and Harry Bruno's 1943 prediction?

   b. Use appropriate terminology to comment on what you think the chances are of either of these predictions coming true.

Check your answers on pages 70 and 71 in the Appendix.

The following time line lists predictions made by famous people about the possibility of space travel. It also lists actual events that happened. Use this time line to answer questions 7 to 10.

- **1865:** Jules Verne says, "We shall one day travel to the moon, the planets, and the stars with the same facility, rapidity, and certainty as we now make the ocean voyage from Liverpool to New York."

- **1929:** British scientist Bickerton says, “This foolish idea of shooting at the moon is an example of the absurd length to which vicious specialization will carry scientists.”

- **1930:** Lord Birkenhead says, “By 2030 the first preparations for the first attempt to reach Mars may perhaps be under consideration.”

- **1932:** U.S. astronomer F. R. Moulton says, “There is no hope for the fanciful idea of reaching the moon because of insurmountable barriers to escaping the earth’s gravity.”

- **1936:** British astronomer Sir Richard van der Riet Wooley says, “The whole procedure [of shooting rockets into space] . . . presents difficulties of so fundamental a nature that we are forced to dismiss the notion as essentially impracticable.”

- **1955:** Dr. Wernher von Braun says, “If we were to start today on an organized and well-supported space program I believe a practical passenger rocket can be built and tested within ten years.”

- **September 1957:** British Astronomer Royal, Sir H. Spencer Jones, says, “Space travel is bunk.”

- **October 1957:** The Russians launched *Sputnik*, the first artificial satellite.

- **1959:** Russian *Luna 2* became the first man-made object to hit the moon.

- **1961:** Cosmonaut Yuri Gagarin became the first man in space, orbiting Earth for 108 min.

- **1969:** American astronauts were the first humans to land on the moon.

- **1969:** Dr. Wernher von Braun says, “By the year 2000 we will undoubtedly have a sizable operation on the moon, we will have achieved a manned Mars landing, and it’s entirely possible we will have flown with men to the outer planets.”

- **1971:** *Mariner 9* (NASA) became Mars' first artificial satellite.

- **1971:** *Mars 3* (USSR) lander achieved the first soft landing on Mars.

- **1996:** Burt Rutan says, "Ten years from now, we will have space tourism."

- **2001:** Dennis Tito became the first space tourist.

- **2001:** NASA announced that a human space explorer will be able to set foot on Mars no later than 2020 and visit other planets of the solar system in the following decades.

7. a. Which one of the events predicted to be impossible actually happened? Explain.

b. Which event predicted to be impossible was disproved most quickly? Explain.

8. Which of the events predicted to be likely to happen actually happened earlier than predicted? Explain.

9. a. Which of the events predicted to be likely to happen has not yet happened?

b. Do they seem likely to happen in the near future? Explain.

10. Predict an event that you think is likely to happen. Tell when you think it will occur, and explain your thinking.

Check your answers on pages 71 and 72 in the Appendix.

Activity 2

Today you will investigate how probability is used to make predictions.

In Activity 1 you saw that some predictions are based on the opinions of knowledgeable people. When the probability of an event happening is based on mathematical calculations, it is known as **theoretical probability**.

Theoretical probability tells you what is expected to happen, based on computation.

Meteorologists use mathematical formulas based on past and present weather conditions to make their forecasts. When you turn on your radio in the morning, you may hear the chance of rain or snow expressed as a probability using percent. Due to the complexity of atmospheric conditions, you also know that predictions about the weather are often incorrect.

1. The following descriptions are based on a table prepared by Environment Canada, Canada's national weather service, as a user's guide to the probability of precipitation.

   Copy and complete the table in your notebook by writing a different percent for each description. Express the percents in multiples of 10, from 0% to 100%. A few are done for you to get you started.

Description of the Chance of Precipitation	Percent
Rain or snow is very likely.	
It's 50-50 on whether you get precipitation or not.	
If you go ahead with your outdoor plans, keep an eye on the weather.	70%
Consider the effect of precipitation on your plans for outdoor activities. The chance for no precipitation is only 3 in 10!	
Want to water your lawn? The odds are favourable that Mother Nature might give you some help.	
No precipitation is expected even though it may be cloudy.	
The occurrence of precipitation is a near certainty.	
Little likelihood of rain or snow; only 1 chance in 10.	
An umbrella is recommended. Consider alternate plans for outdoor activities that are susceptible to rain. It's not a good day to pave the driveway.	40%
Precipitation is a certainty.	
No precipitation is expected.	20%

2. Why are the percents given as multiples of 10, rather than exact numbers such as 47?

3. Does a 40% probability of rain mean it will rain continuously for 40% of the day? Explain.

4. Explain how the probability of precipitation might be used in making the following kinds of decisions.

   a. business decisions
   b. personal decisions

Check your answers on page 72 in the Appendix.

If you have access to the Internet, you can find this table at the following website:

http://www.msc-smc.ec.gc.ca/cd/brochures/probability_e.cfm

Two American professors developed a theoretical model to predict how many medals various nations would win at the 2000 Summer Olympics in Sydney, Australia. They based their mathematical formula on population, per-capita income, government funding, past Olympic performance, and host country advantage. In total, their predictions were reported to be 96% accurate. Use this information to answer questions 5 to 9.

5. Use probability words to describe the likelihood that this model could be used to make accurate predictions for future Olympic Games.

6. Why might a country's population influence how many medals that country wins?

7. Why might government funding affect a country's chance of winning?

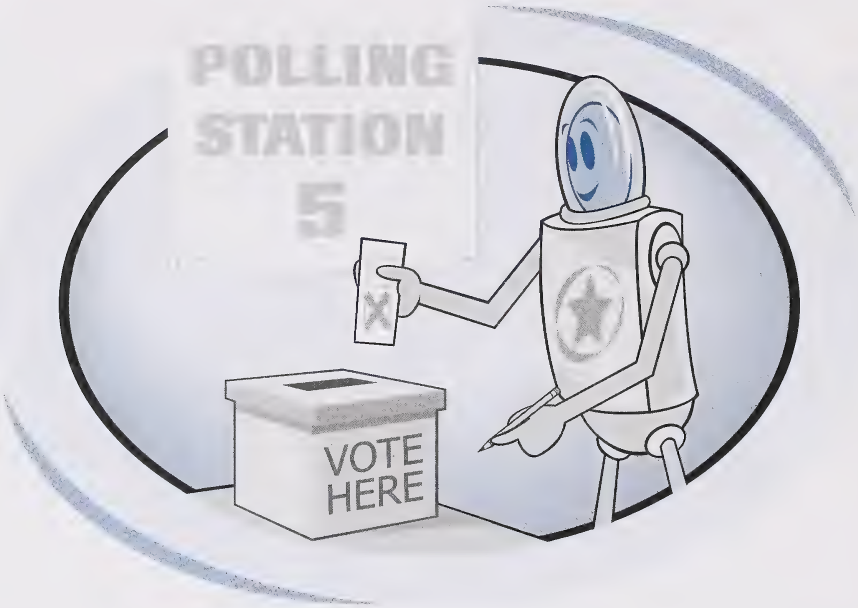
8. Do you think it would be easier to predict who will win a gold medal in a diving event or in high jumping? Explain.

9. How is predicting Olympic medal winnings similar to predicting the weather?

Check your answers on page 73 in the Appendix.

Activity 3

Today you will use equivalent fractions and probabilities to make predictions.



In Module 8 you saw how samples are used to predict results that can be used to make decisions.

Predictions are statements about what is expected to happen.

Predictions in everyday life may be expressed as percents, fractions, or decimals. For example, before an election, predictions are made about how many eligible voters will actually vote. In Alberta, the expected voter turnout for a provincial election is about 50% of the eligible voters. If about 2 000 000 Albertans are eligible to vote, then equivalent fractions can be used to find the number of people that are expected to vote.

$$50\% = \frac{50}{100} = \frac{1}{2} = \frac{1\ 000\ 000}{2\ 000\ 000}$$

This means that about 1 000 000 Albertans are expected to vote in a provincial election.

Look at a second example.

Example

In Edmonton, about 500 000 people were eligible to vote in the 2001 civic election.

a. If the voter turnout was predicted to be 40%, find the number of people that were expected to vote.

b. If 300 000 people voted, what percent of all eligible voters actually voted?

Solution

a. The voter turnout was predicted to be 40%.

$$\begin{aligned} 40\% &= \frac{40}{100} \\ &= \frac{2}{5} \\ &= \frac{200\,000}{500\,000} \begin{array}{l} \longleftarrow \text{Number expected to vote} \\ \longleftarrow \text{Total number of voters} \end{array} \end{aligned}$$

About 200 000 were expected to vote.

b. About 300 000 out of 500 000 people voted.

$$\begin{aligned} \frac{300\,000}{500\,000} &= \frac{3}{5} \\ &= \frac{6}{10} \\ &= \frac{60}{100} \\ &= 60\% \end{aligned}$$

The actual turnout was 60%.

For questions 1 to 8, use equivalent fractions to find the answers. Show your work.

The population of Canada is about 30 000 000. Two separate polls conducted in January 2002 showed the following statistics:

- 20% of Canadians surveyed saw the economy as their main concern.
- Only half of Canadians know the name of the country's first prime minister.

Use this information to answer questions 1 and 2.

1. About how many Canadians saw the economy as their main concern?

2. About how many Canadians know the name of Canada's first prime minister?

Check your answers on page 73 in the Appendix.

3. Using the results of a survey, a supermarket manager predicted that, in the year ahead, 8 out of every 10 floral sales would be impulse purchases.

a. If about 100 floral sales are made each day, how many of these would be expected to be impulse purchases?

b. Does this mean the manager can expect that 80% of his total profits from floral sales would be from impulse purchases? Explain.

4. At a factory, a sample of 100 out of every 10 000 light bulbs is tested. If 2 light bulbs out of a sample of 100 are faulty, how many faulty light bulbs might be expected in the population of 10 000 light bulbs?

Check your answers on page 74 in the Appendix.

The owner of Ho Mah Ting's Pizzeria predicted his sales each week as shown in the following tables. Use this information to answer questions 5 to 7.

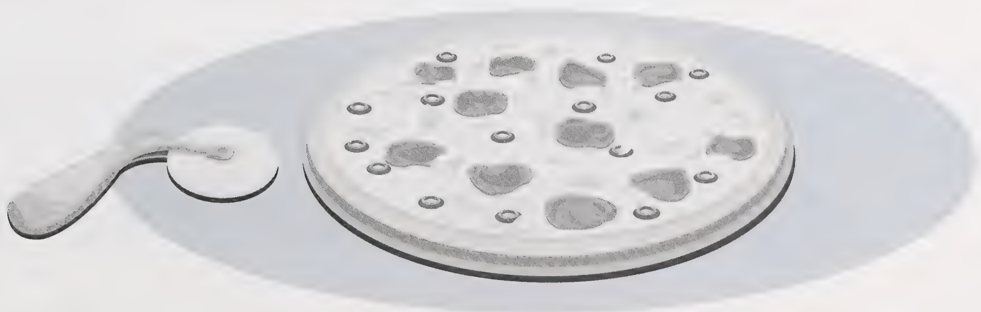
These questions are optional. Do them if you need more practice making predictions based on probabilities.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Percent of All Pizzas Sold	9%	10%	12%	13%	23%	21%	12%

Type of Pizza	Percent of All Pizzas Sold
Pepperoni and Mushroom	30%
Hawaiian	27%
Ground Beef and Onion	18%
Bacon and Tomatoes	7%
Everything	6%
Plain Cheese	5%
Vegetarian	4%
Other	3%

5. If he sold 1000 pizzas every week, how many would he expect to sell each Friday?

6. If he sold 500 pizzas every week, how many would he expect to be Hawaiian?



7. About how many Hawaiian pizzas might he expect to sell on Tuesday?

8. Jill's Jellies have a mass of about 1 g each. Jack found there were 19 green jellies in a 125-g bag. He predicted there would be about 150 green jellies in a 1-kg bag. Explain whether or not you agree with Jack.

Check your answer on page 74 in the Appendix.

Challenge Activity

The six faces of a die are numbered 1 through 6. Kylee tossed the die 36 times and recorded each outcome. Find the expected sum of all the outcomes.

Check your answer on page 75 in the Appendix.

Conclusion

In this lesson you reviewed how probability vocabulary is used when making predictions in everyday life. You saw that events do not always occur in the way they are predicted. You used percents and equivalent fractions to express probabilities.

During a long cold winter, do you yearn for warmer weather and wonder how long it will be before spring arrives? Do you pay any attention to the predictions made on Groundhog Day or contained in almanacs?

If you have an almanac at home, you may find it interesting to leaf through it and look at its long-range forecasts for the weather. Read the predictions made for the current month and see how accurate they were. Did you know that these predictions are based on long-term averages for the weather and probabilities? It would be a strange forecast indeed if they didn't predict warmer weather for March or April!

Turn to Assignment Booklet 9A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 9A until you have completed the entire booklet.

Lesson 2

Outcomes and Chance

If you flip a fair coin, you would expect the coin to turn up heads half the time and tails half the time. After 100 tosses you would expect to see 50 heads and 50 tails. But is that what happens in practice?

You may want to conduct an experiment. Flip a coin 100 times and record the number of heads and the number of tails. If you performed this experiment once, it is unlikely you will see the same number of heads and tails. There may be more heads than tails! And, if you repeat the experiment, you may see more tails than heads!

In this lesson you will investigate the relationship between theoretical probability and experimental results. You will calculate both the theoretical and **experimental probability** of events based on possible outcomes. You will see that different results may occur when you repeat an experiment.

Activity 1

Today you will estimate probabilities.


NASA

Space missions require teamwork as there are many specialized tasks to be done.

An astronaut's background and training determines the roles for which he or she is qualified. Space Shuttle missions require the following crew members:

- a commander to oversee the entire operation
- a pilot to complete launch and landing sequences
- mission specialists to oversee various tasks, such as housekeeping, communications with Mission Control, exercise programs, and computer operations

The chance that an astronaut will be chosen for a space mission depends on the number of astronauts needed in the specialized roles for which he or she has been trained and the number of astronauts available for those specialized roles.

For example, Commander Bilinski wants to find out how likely it is that she will be part of the next mission. She knows it is possible that she will be chosen. Yet, she is aware that it isn't a certainty because there are other commanders available for the mission.

When the probability of an event is expressed as a fraction, an impossible event is represented by 0 and a certain event is represented by 1.

Suppose that one commander, one pilot, and two mission specialists are needed for the mission. The following astronauts are available and equally likely to be chosen for their specialty:

Commanders: Bilinski, Ho, and Evans

Pilots: Hardy and Goldberg

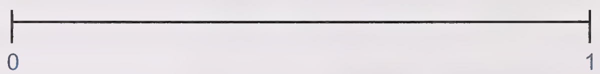
Mission Specialists: Kasparov, Yano, and McHugh

Use this information to answer questions 1 to 6.

1. a. Explain why the chance of Bilinski being chosen as commander is 1 out of 3.

b. Write the probability of Bilinski being chosen as commander as a fraction.

2. Estimate and place a mark on a number line like the following to show the probability of a particular commander being chosen. Explain your answer.



3. a. Write, as a fraction, the probability of a particular pilot being chosen. Explain.

   b. Place a mark on a number line to show the probability.

Check your answers on pages 75 and 76 in the Appendix.

To find the chance of any particular astronaut being chosen for a mission, compare the number of crews that could be formed with that particular astronaut as a member (number of **favourable outcomes**) to the total number of different crews that can be formed (number of **possible outcomes**).

4. a. List all the possible outcomes for picking pairs of mission specialists if the order they are written in the pair doesn't matter.

   b. How many possible outcomes are there?

   c. In how many of the outcomes does the name of each mission specialist appear?

   d. Using your answers from questions 4.b. and 4.c., write the probability of a particular mission specialist being chosen as a fraction. Explain.

   e. Estimate and place a mark on a number line to show this probability.

Finding the likelihood that something will happen by comparing favourable outcomes to possible outcomes is called theoretical probability. This method is summarized as follows:

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

5. a. List all the possible outcomes for a commander-pilot team.

   b. How many possible outcomes are there?

c. Using your answer to question 5.b., write, as a fraction, the probability of Evans and Hardy being chosen as the commander-pilot team. Explain.

d. Estimate and place a mark on a number line to show this probability.

Questions 6 to 10 are optional. Do the questions if you need more practice determining theoretical probabilities.

6. There are 18 possible ways to choose the crew. Study the pattern and copy and complete the table in your notebook.

	Commander	Pilot	Mission Specialists
Crew 1	Bilinski	Hardy	Kasparov and Yano
Crew 2	Bilinski	Hardy	Kasparov and McHugh
Crew 3	Bilinski	Hardy	Yano and McHugh
Crew 4	Bilinski	Goldberg	Kasparov and Yano
Crew 5	Bilinski	Goldberg	Kasparov and McHugh
Crew 6	Bilinski	Goldberg	Yano and McHugh
Crew 7	Ho	Hardy	Kasparov and Yano
Crew 8	Ho	Hardy	
Crew 9	Ho	Hardy	
Crew 10	Ho	Goldberg	
Crew 11	Ho		
Crew 12	Ho		
Crew 13	Evans		
Crew 14			
Crew 15			
Crew 16			
Crew 17			
Crew 18			

7. a. Write, as a fraction, the probability of a particular crew being chosen. Explain.

   b. Estimate and place a mark on a number line to show this probability.

8. Suppose that Hardy became unavailable for the mission.

   a. Write the probability of Goldberg being chosen. Explain.
   b. Place a mark on a number line to show this probability.

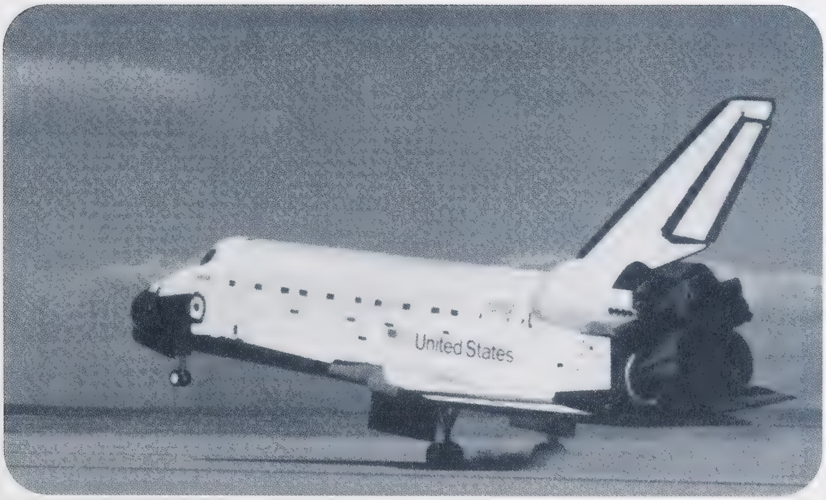
9. Suppose an additional commander, Smith, became available for the mission.

   a. Write, as a fraction, the probability of a particular commander being chosen. Explain.

   b. Estimate and place a mark on a number line to show this probability.

10. Use the original list of available astronauts. Compare the likelihood of Evans being chosen for the crew with the likelihood of Kasparov being chosen for the crew. Explain.

Check your answers on pages 76 to 78 in the Appendix.



Activity 2

Today you will solve problems using probabilities.

When you travel by car, do you ever predict whether a particular traffic light on your route will be red, green, or yellow when you reach it? Traffic lights at an intersection are programmed to change colour according to a timed cycle.

The following tables show the timed cycles for some traffic lights. Use this information to answer questions 1 to 6.

INTERSECTION 1: SALE WAY AND RUE DE PELLETIER

Light Colour	Daytime Hours		Nighttime Hours	
	Flow of Traffic		Flow of Traffic	
	Along Sale	Along Pelletier	Along Sale	Along Pelletier
Green	20 s	20 s	25 s	25 s
Yellow	4 s	4 s	5 s	5 s
Red	24 s	24 s	30 s	30 s
Total Cycle Time	48 s	48 s	60 s	60 s

INTERSECTION 2: QUINN BOULEVARD AND HAUK DRIVE

	Both Day and Night	
	Flow of Traffic	
Light Colour	Along Quinn	Along Hauk
Green	20 s	10 s
Yellow	4 s	4 s
Red	16 s	26 s
Total Cycle Time	40 s	40 s

The probability (P) of a traffic light being a particular colour can be found as follows:

$$P = \frac{\text{number of seconds in the cycle the light is a particular colour}}{\text{total number of seconds in the cycle}}$$

1. **a.** For how many seconds in one cycle is the light green along Pelletier at Intersection 1 during daytime hours?

   b. How many seconds long is one total cycle at Sale and Pelletier during daytime hours?

   c. Use your answers to write the probability, as a fraction, of the light being green along Pelletier at Intersection 1 at noon.

   d. Divide the numerator and denominator of your fraction by the GCF to write an equivalent fraction (in simplest terms) for the probability in question 1.c.

2. **a.** Write, as a fraction, the probability of the light at Intersection 1 being red during the daytime. Simplify your fraction. Show your work.

   b. Write, as a fraction, the probability of the light at Intersection 1 being yellow during the daytime. Simplify your fraction. Show your work.

3. Write, as a fraction, and then simplify the probability of having each of the following colours if you are driving through Intersection 1 at night.

   a. green
   b. red
   c. yellow

4. How are the daytime and the nighttime traffic-light cycles for Intersection 1 the same? How are they different?

5. Write, as a fraction, and then simplify the probability of Intersection 2 being the following colours along Quinn.

   a. green
   b. red
   c. yellow

6. Write, as a fraction, and then simplify the probability of Intersection 2 being the following colours along Hauk.

   a. green
   b. red
   c. yellow

Questions 7 to 9 are optional. Do these questions if you want a challenge.

7. At Intersection 1, the light is red along Pelletier for the same length of time as it is either green or yellow along Sale, and vice versa.

   Daytime $20 + 4 = 24$; Nighttime $25 + 5 = 30$

   This means that at Intersection 1, the light turns green for one flow of traffic at the exact same second that the light turns red for the other flow of traffic. (One traffic flow must be stopped when the other traffic flow is going.)

   a. Explain the programming feature on which Intersection 2 is based.

   b. Explain which intersection has been programmed with the better feature.

8. For which of the different flows of traffic shown in the tables is the probability of the light being green

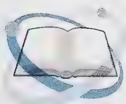
   a. the greatest?
   b. the least?

9. What conditions do you think are considered when decisions are made about setting light-cycle patterns at an intersection?

Check your answers on pages 78 and 79 in the Appendix.

Sharing Time

Now it's time to show your instructor what you have been learning.



Turn to page 106 of the Practice and Homework Book and complete questions 1 to 6.

Discuss your answers with your home instructor.

Activity 3

Today you will compare theoretical and experimental probabilities.

A restaurant uses the game card shown below. Every time you buy a kid's meal, you toss a die and your card is stamped in a square above the number you toss.

For example, the game card shows the results of tossing 2, 4, 6, 2, and 3 for the first five meals. When one column is completely filled (six stamps), you win that treat. Then, you start over with a new game card.

	Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
		☺				
		☺	☺	☺		☺
Number tossed:	1	2	3	4	5	6

1. The outcome of each game card is the treat won. List all the possible outcomes.

2. Write, as a fraction, the theoretical probability for each possible outcome.

3. a. How many of each prize would you expect to win after completing six cards? Explain.

   b. Is what you predicted in the previous question likely to happen? Explain.

Check your answers on pages 79 and 80 in the Appendix.

4. Use the game cards from the Appendix to play the game six times. For each card, keep tossing a die and marking a square in the column above the number tossed until a column is filled.

5. Tally the results for each card in a table like the following. Circle the tallies of the treat won.

	Tallies					
Treat	**Card 1**	**Card 2**	**Card 3**	**Card 4**	**Card 5**	**Card 6**
Ice Cream						
Soft Drink						
Hot Dog						
Burger						
Fries						
Muffin						

6. a. Use your answer from question 5 to write the experimental probability for each possible outcome (the six different treats) as a fraction.

b. Why might your experimental results be different from the theoretical probability? Explain what happened in your case.

Check your answers on pages 80 and 81 in the Appendix.

Questions 7 to 9 are optional. Do these questions to extend your skills.

7. Write the frequencies for the different treats from each of the cards in the appropriate cells in a table like the following. (To do this, count the tallies in the corresponding cells from the table of answers in question 5.) Then, complete the total frequency column and row by finding the sum of the frequencies for the corresponding columns and rows. Once you have done this, use the completed table to answer questions 8 to 10.

	Frequencies						
Treat	**Card 1**	**Card 2**	**Card 3**	**Card 4**	**Card 5**	**Card 6**	**Total Frequency**
Ice Cream							
Soft Drink							
Hot Dog							
Burger							
Fries							
Muffin							
Total Frequency							

8. a. Each tally mark shown in the table in question 5 represents one toss of the die after buying one kids' meal. Which cell of the table in question 7 tells how many kids' meals were purchased to win six prizes?

b. How were you able to tell if a mistake was made when adding columns or rows of frequencies in the table in question 7?

c. How many kids' meals were purchased to win six prizes?

d. Use your answer to question 8.c. If that number of kids' meals were purchased, how many times would you expect any one of the treats to have been rolled? Explain.

e. How does the experimental total frequency for each treat compare with your answer to question 8.d.?

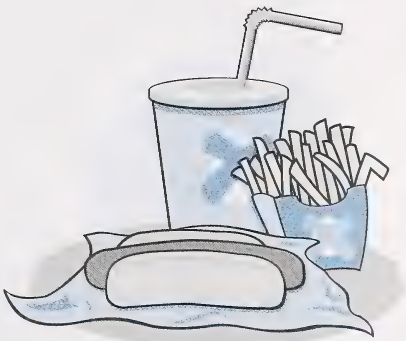
9. Use the total frequencies for each card (the bottom row of the table from question 7).

a. What was the least number of kids' meals you needed to buy in order to win a treat?

b. What was the greatest number of kids' meals you needed to buy to in order to win a treat?

c. What is the least possible number of kids' meals anyone would need to buy to win a treat? Explain.

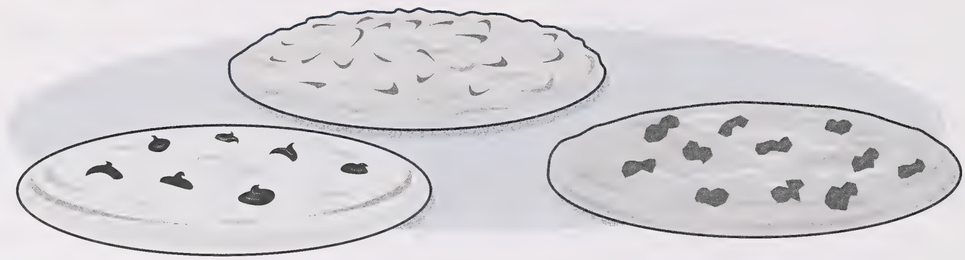
d. What is the greatest possible number of kids' meals anyone would need to buy to win a treat? Explain.



e. Based on your experimental results from question 8.c., predict how many kids' meals you might need to buy in order to win another prize. Explain.

Check your answers on pages 82 and 83 in the Appendix.

Challenge Activity

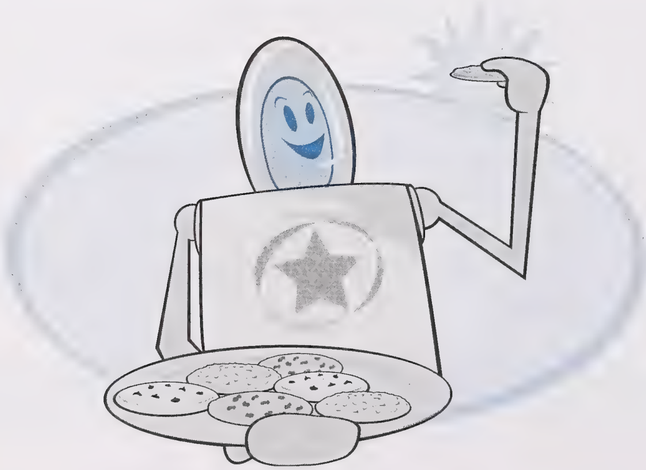


Greg's mom baked 3 different kinds of cookies: oatmeal, chocolate chip, and raisin. She stored them randomly in a large cookie jar. Greg took 4 cookies out of the cookie jar without looking.

1. Organize, label, and complete a table to list all the possible combinations of cookies that Greg could have chosen. (The order the cookies were picked in isn't important. What matters is how many of any particular type of cookie Greg chose).

2. How many different combinations are possible?

Check your answers on page 84 in the Appendix.



Conclusion

In this lesson you investigated the relationship between theoretical probability and experimental results. You calculated both the theoretical and experimental probability of events, based on possible outcomes. You saw that different results may occur when you repeat an experiment.

The prize distribution in lotteries is based on the total amount wagered and the theoretical probability of winning a prize. Many people dream of winning, but did you know that for a popular lottery in which the grand-prize winner must select the six winning numbers from 49 possible numbers, the theoretical probability is only 1 chance in 13 983 816?

Think of a swimming pool that is 5 m wide by 10 m long filled to a depth of approximately 30 cm with 13 983 815 red jellybeans. Mixed in these jellybeans is a single black jellybean. You are blindfolded and asked to wade into the pool and select just one jellybean. Your chances of selecting the black jellybean are the same as winning the lottery!

Turn to Assignment Booklet 9A and complete the Lesson 2 Assignment.

When you are done, send Assignment Booklet 9A to your teacher to be marked.

Lesson 3

Games and Fairness

The French mathematician Blaise Pascal (1623–1662) pioneered the study of probability. He based his study on letters he wrote in the summer of 1654 to another mathematician, Pierre de Fermat, about the outcomes of games of chance. In particular, Pascal and Fermat discussed games involving dice and how the stakes in those games should be divided fairly among the players if the games were interrupted before they could be completed.

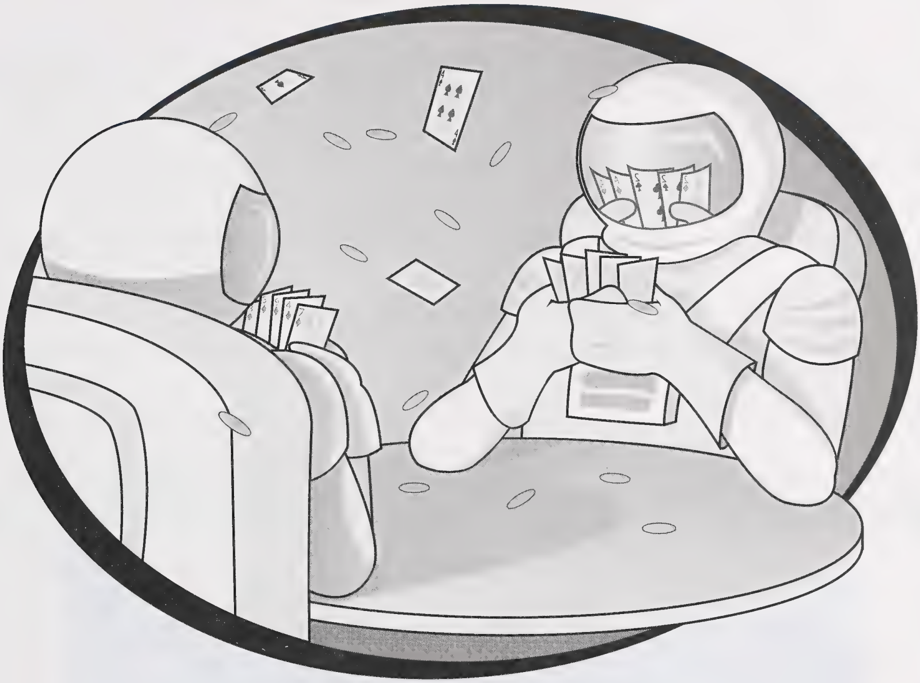
If you have access to the Internet you can find out more about Pascal at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Pascal.html

In this lesson you will explore how probability is involved in games, and you will investigate what is meant by a **fair game**. You will use a variety of dice that have different numbers of faces and compare theoretical outcomes with experimental outcomes.

Activity 1

Today you will explore what is meant by a fair game.



Social activities, such as playing games, are an important part of life in space.

You are likely familiar with the game tick-tack-toe. Three-in-a-row games are found in many other cultures. Achi in Ghana, Derrah in Nigeria, and Seega in Egypt are other similar forms of the game.

Commander Claire made a three-in-a-row game she called Planets Aligned for astronauts to play on the Space Shuttle. Her game consists of the following pieces:

- nine planet cards (found in the Appendix)
- nine double-sided chips (found in the Appendix)
- a game board (found in the Appendix)

In the game of Planets Aligned, you try to get three coloured chips in a row on the game board as often as possible. Here is how you play the game:

- One player uses the blue side of the chips. The other player uses the white side of the chips.

- A game consists of ten rounds. For each round, play is as follows:
  - The nine planet cards are shuffled and the players take turns going first.
  - The players alternate taking the top planet card and then placing their chip on that planet on the board.
  - The first player to get three colour chips in a row wins that round and gets 1 point.
  - If all nine cards are played and nobody gets three in a row, the round is recorded as a draw. (It still counts as a round played, but nobody wins it.)

- The winner of the game is the player with the most points at the end of the tenth round (the person who has won the most rounds).

Remove the game board, cards, and chips from the Appendix. Cut out the cards and chips. If possible, play the game with another person. Otherwise, you can play it by yourself taking turns using alternating colours.

1. Play one complete game (ten rounds). Record your results on a tally chart.

2. List the different possible outcomes for one round of the game. (How could any one round end?)

3. List the different possible outcomes for one completed game. (How could any one game end?)

4. Explain what is meant when a game is said to be "fair."

5. Theoretically speaking, do you think Planets Aligned is a fair game? Explain.

6. a. What is the least possible number of cards that must be drawn in all in order for a player to win a round?

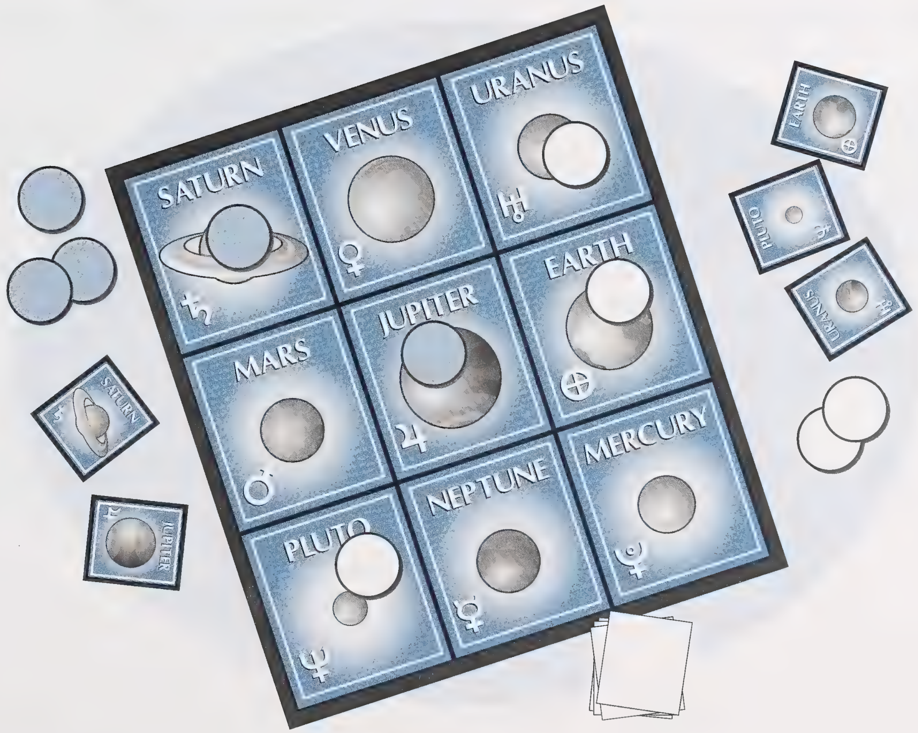
   b. Explain your answer to question 6.a. by drawing a diagram that shows a possible arrangement of chips on the game board.

7. Sometimes the winning chip completes two rows at once, so the astronauts agreed on another rule: When a double win happens, the winner will receive 2 points for that round. Keep playing the game until a round ends with a double win. Draw a diagram that shows a possible arrangement of chips on the game board just before the winning chip makes a double win.

8. Do you think the double-win rule affects the fairness of the game? Explain.

9. Can you become more skilful at this game if you play it often? Explain.

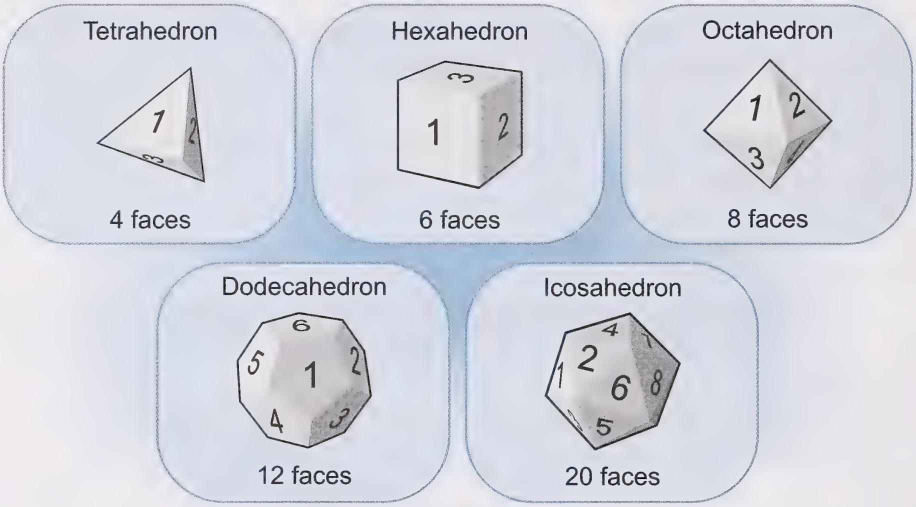
Check your answers on pages 84 to 86 in the Appendix.



Activity 2

Today you will investigate probabilities involving dice.

Felipa's aunt gave her a board game that has an interesting set of dice. The faces of each die are numbered with consecutive counting numbers, beginning with 1. These are shown in the following picture.



The polyhedra in this set are known as the **Platonic solids**. Only five different polyhedra belong to this set with the following special features:

- All the faces are identical (same size and shape) regular polygons.
- The same number of faces meet at each vertex.

Use this information to answer questions 1 to 6.

1. Copy and complete the following table in your notebook.

Polyhedron	Shape of Faces	Consecutive Numbers Written on the Faces
		1, 2, 3, 4, 5, 6, 7, 8
Tetrahedron	equilateral triangles	
		1, 2, 3, 4, 5, 6
Icosahedron		
	regular pentagons	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

2. Explain why Platonic solids are used for dice.

3. Why is the hexahedron (cube) likely the most popular shape used for a die?

Check your answers on page 86 in the Appendix.

4. Copy and complete the following table in your notebook by writing each probability as a fraction. Remember, 1 is not a prime number.

		Probability of Favourable Outcome			
		Multiple of 5	Number Less Than 6	Multiple of 4	Prime Number
Die Used	Tetrahedron				
	Hexahedron				
	Octahedron				
	Dodecahedron				
	Icosahedron				

Use the table you completed in question 4 to answer questions 5 to 9.

5. Identify the Platonic die (or dice) for which

a. it is impossible to toss a multiple of 5
b. you have a 50% chance of tossing a prime number
c. you are very likely to toss a number less than 6

Check your answers on page 87 in the Appendix.

6. Five people are each using a different Platonic die to play a game. For each round, they all toss their dice at once. Those who toss a multiple of 4 win a point. The player with the most points after 10 rounds wins the game.

a. Is it possible for any particular player to win the game? Explain.
b. Is this a fair game? Explain.

7. Two people are each using one dodecahedral die to play the following game. Player A gets a point if she tosses a number less than 6. Player B gets a point if he tosses a prime number. The winner of the game is the player with the most points after 12 rolls. Is this a fair game? Explain.

8. Two people are each using an octahedral die to play the following game. Player A gets 2 points if he tosses a multiple of 5. Player B gets 1 point if she tosses a multiple of 4. The winner of the game is the player with the most points after 8 rolls. Is this a fair game? Explain.

9. For which Platonic die are you least likely to roll a prime number? Explain.

Check your answers on pages 87 and 88 in the Appendix.

Activity 3

Today you will make some dice, toss them, and compare the theoretical (expected) results with the experimental results.

In Activity 2 you explored the theoretical probability that particular outcomes would occur when you tossed various dice.

Turn to page 256 in your textbook to Theoretical Probability. The pictures show the nets for making a tetrahedron, a cube, an octahedron, and a square pyramid. (See page xx in the Appendix for the nets of these polyhedra.) Cut out the nets and use them to answer questions 1 to 8. **Note:** Don't put the nets together until instructed to do so.

1. Put numbers on the faces of the cube so the outcome listed has the given theoretical probability.

    - The probability of tossing 5 is $\frac{1}{6}$.
    - The probability of tossing 4 is $\frac{1}{2}$.
    - The probability of tossing 3 and the probability of tossing 6 are equally likely.

2. Fold and tape the cube net from question 1 to make a cube.

   a. Toss the cube 30 times and record the results in a tally chart.

   b. Use equivalent fractions to calculate the expected results for each possible outcome.

   c. How do your answers to question 2.a. and 2.b. compare?

3. Colour the faces of the tetrahedron so that when you toss it

   - the outcomes blue and yellow are equally likely
   - the outcome red has a theoretical probability of 50%

4. Fold and tape your net from question 3 to make a tetrahedron.

   a. Write, as a fraction, the theoretical probability of each possible outcome.

   b. Suppose you were to toss the die 32 times. Write, as a fraction, the expected result for each outcome. Show your work.

   c. Toss the tetrahedron 32 times and record the experimental results in a tally chart.

   d. How do your answers to question 4.b. and 4.c. compare?

Check your answers on pages 88 to 90 in the Appendix.

5. Mark each face of the octahedron with either an **X** or an **O** so that the theoretical probability of tossing an **O** is three times as likely as the probability of tossing an **X**.

6. Fold and tape your net from question 5 to make an octahedron.

   a. Write, as a fraction, the theoretical probability of each outcome.

   b. Suppose you were to toss the die 40 times. Find the expected frequency for each outcome. Show your work using equivalent fractions.

c. Toss the octahedron 40 times and record the experimental results in a tally chart.

d. How do your answers to question 6.b. and 6.c. compare?

7. Mark each of the faces of the square pyramid with a different number from 1 to 5. Fold and tape your net to make a square pyramid.

   a. If you toss the pyramidal die, what are all the possible outcomes (faces on which it might land)? Explain whether or not the possible outcomes are equally likely.

   b. Suppose you were to toss the pyramidal die 20 times. Use your answer from question 7.a. to guess the frequency for each outcome.

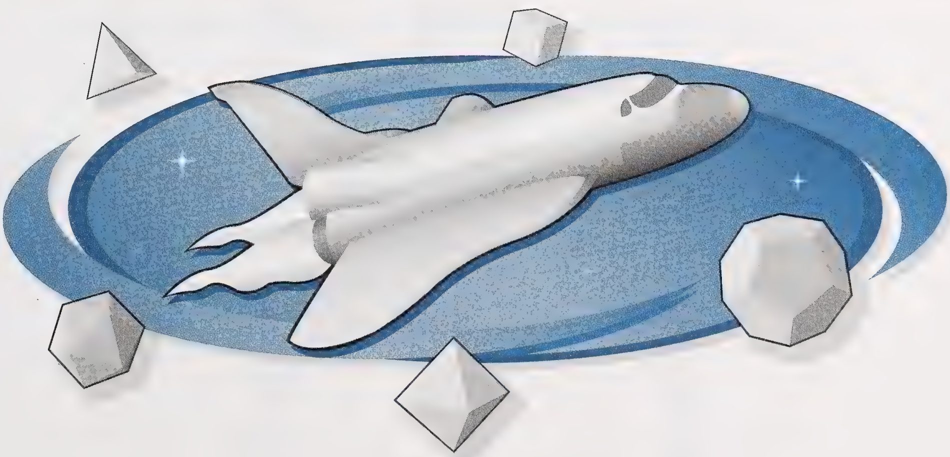
   c. Toss the pyramidal die 20 times and record the experimental results in a tally chart.

   d. How do your answers to question 7.b. and 7.c. compare?

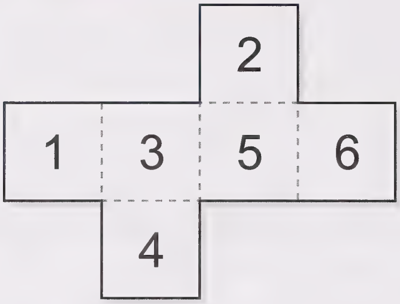
8. a. If any polyhedron can be used as a die, how do the sizes and shapes of its faces affect the probabilities of its outcomes?

   b. What shape of polyhedron should be used as a die?

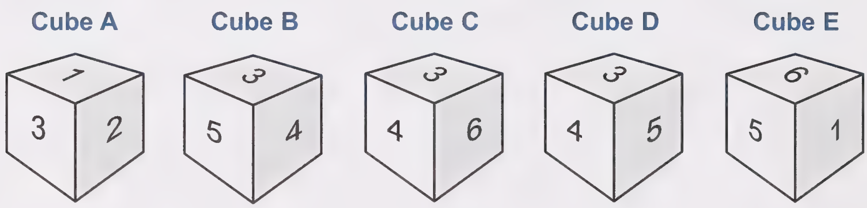
Check your answers on pages 90 to 92 in the Appendix.



Challenge Activity



1. Which of the following cubes are made from the net shown above? (Don't worry about the direction that the numbers are written; only that they are on the correct faces.)



2. **a.** Which number will be on the cube's bottom face?

   b. How can you tell the answer to question 2.a. just by looking at the net shown above?

3. Which number will be on its hidden face to the left?

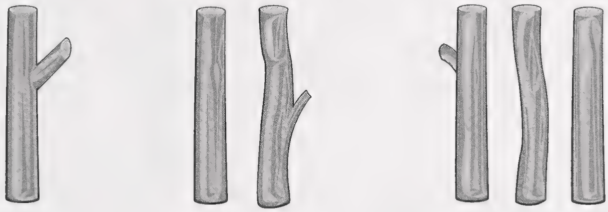
4. Which number will be on its hidden face to the right?

Check your answers on pages 92 and 93 in the Appendix.

Conclusion

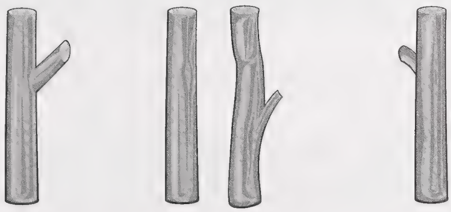
In this lesson you explored how probability is involved in games and you investigated what is meant by a fair game. You used a variety of dice that have different numbers of faces and compared theoretical outcomes with experimental outcomes.

One game that is definitely not fair involves two players and three sets of sticks. There is one stick in the first set, two sticks in the second set, and three sticks in the third set.



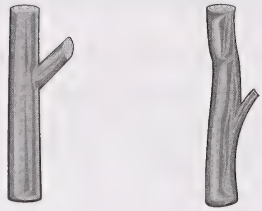
The players take turns removing sticks. Any number of sticks can be removed as long as they belong to the same set and as long as they are side by side. The player who removes the last remaining stick loses. As it turns out, the first player will always lose as long as the second player does not make any mistakes.

For example, suppose the first player removes two sticks from the third set.



To counter, the second player removes one stick from the second set.

Now, the first player can only remove one stick.



The second player removes a stick and the first player is forced to remove the last stick and loses the game.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Play the game described in the Conclusion with your home instructor. Discuss with your home instructor whether this game is fair or not.

Turn to Assignment Booklet 9B and complete the Lesson 3 Assignment.

Keep Assignment Booklet 9B until you have completed the entire booklet.

Module Summary

In this module you reviewed how the vocabulary of probability is used when making predictions in everyday life. You saw that events do not always occur in the way they are predicted. You used percents and equivalent fractions to express probabilities.

Also, you investigated the relationship between theoretical probability and experimental results. You calculated and compared theoretical and experimental probability of events based on possible outcomes. You saw that different results may occur when you repeat an experiment. You explored how probability is involved in games and you investigated what is meant by a fair game. You used a variety of dice that have different numbers of faces and investigated the outcomes when tossing them.

The probability you explored in this module is fundamental to a variety of everyday applications. For example, the insurance industry uses probability to determine the premiums it charges various classes of drivers. Young male drivers are charged high premiums because experience shows that this group is likely to be involved in more accidents than most other drivers. Do you think differences in automobile insurance based on age or sex are justified?

Turn to Assignment Booklet 9B and complete the Numbers in the News project.

When you are done, send Assignment Booklet 9B to your teacher to be marked.

Keystrokes

Take out your calculator and complete the following exercises. They will help you review some of the ideas you have learned in Module 9.

Funky Feature: Sum Total

The following table shows all 36 possible outcomes when you toss two dice and add the numbers.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The sum of all the sums in the table is 252.

To play the following game, you will need a calculator for each player and two dice.

- Players take turns tossing the dice. Each player makes 36 tosses in all.
- For each turn, a player enters 1000 plus the sum of the two dice and presses the plus key. The thousands keep track of the number of tosses and the last three digits keep a running total. For example, if your calculator shows 24171, then you have completed 24 tosses and your running total is 171.

- After 36 tosses, the player whose total sum is closest to 252 wins. For example, if a player tosses 4 and 7 on the first turn, 5 and 5 on the second turn, 1 and 3 on the third turn, . . . , and 3 and 6 on the thirty-sixth turn, he or she would enter the keystrokes as follows:

Keystrokes	1011	+	1010	+	1004	+	. . .	1009	+
Display	1011	1011	1010	2021	1004	3025	. . .	1009	36241

Play the game with a partner, if possible. You may also play it as a solitaire game and see how close to 252 you can get.

1. Explain what the five digits in the final display mean.

2. What is the least possible final display? Explain. Is this likely?

3. What is the greatest possible final display? Explain. Is this likely?

4. Is this a fair game? Explain.

5. Can you become more skilful at this game if you play it often? Explain.

Check your answers on page 93 in the Appendix.

Funky Feature: Lose a Million!

To play the following game, you will need a calculator for each player and six dice:

- Players take turns tossing the dice.

- The object of the game is to lose exactly 1 000 000 points.

- Each player enters 1000000 on his or her calculator and presses the minus key.

- For each turn, a player may choose how many dice to toss (one to six), and then the dice are tossed all at once.

- The player lines up all the tossed dice to form a number, enters that number, and presses the minus key. The display tells how many points are left.

- A player who cannot use the tossed dice to make a number that is less than or equal to the display loses that turn.

- The first player to display 0 wins the game. For example, if a player tosses 4, 5, 1, 6, 3, and 5 on the first turn, and chooses to line them up to form 655 431, he or she then enters the following keystrokes:

Keystrokes	1000000	–	655431	–
Display	1000000	1000000	655431	344569

Play the game with a partner, if possible. You may also play it as a solitaire game and see how many tosses it takes to display 0.

6. Is this a fair game? Explain.

7. Describe your strategies for choosing the number of dice to use each turn.

8. Can you become more skilful at this game if you play it often? Explain.

Check your answers on page 93 in the Appendix.

Review

The activities in this lesson will help you review and apply what you learned in Module 9 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.

1. Turn to page 250 of your textbook to Starting Out. Rewrite what each person is saying so that it means the same thing but uses different words than *likely*, *chance*, and *outcome*.

2. Use each mark on the number line only once. Print the letter for each of the following forecasts beneath the mark that best represents the probability that it will happen.

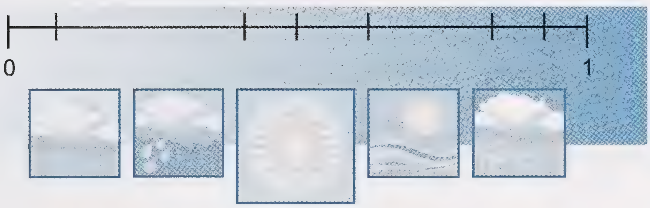
   a. There is a 50% chance that it will snow today.

   b. Conditions are favourable that we will receive scattered snow flurries in the morning.

   c. Precipitation is highly unlikely for today.

   d. Thunderstorms are almost certain for this weekend.

   e. Expect today to be windy and cool.



3. Turn to page 255 of your textbook. Do questions 1 to 3 of Practise Your Skills.

If you need help with questions 1 to 3, look back at Lesson 1, where you learned about probability vocabulary and using equivalent fractions to express probabilities. If you feel you need more practice, do questions 4 and 5.

4. Rewrite each of the following statements using different words for those shown in italics.

   a. My soccer team has a *chance* of winning tomorrow's game.

   b. *Most likely* there will be a large crowd of spectators at tomorrow's soccer game.

   c. The visiting team says we will *never* beat them.

5. Turn to page 262 of your textbook. Do questions 1 to 8 of Practise Your Skills.

Check your answers on pages 94 and 95 in the Appendix.

6. Turn to page 252 of your textbook to Starting Out. Use the information shown to answer the following questions.

   a. Explain how the information about the players shown on the team list is used to make the line-up cards.

   b. Make a different possible line-up card using players from the team list.

   c. Does the schedule of fall games show all possible pairs of teams?

   d. Suppose the first team named in each pair is the home team. How many more games have to be scheduled so that every team hosts every other team once?

7. Turn to page 262 of your textbook. Do question 2 of On Your Own. Explain.

8. Turn to page 270 of your textbook. Do questions 2 and 4 of Skill Bank from This Unit.

If you need help with questions 6 to 8, look back at Lesson 2, where you learned about finding the probability of possible outcomes. If you feel you need more practice, do questions 9 and 10.

9. Turn to page 264 of your textbook to Finding Probabilities for Combinations. How many different ways can you combine a shirt (brown, white, beige, blue) with a pair of pants (brown, black, blue)?

10. Turn to page 291 of your textbook to Skill Bank Looking Back. Use the menu shown in question 4. If you randomly choose a soup and a sandwich, what is the probability you will choose vegetable soup and an egg salad sandwich? Explain.

Check your answers on pages 95 and 96 in the Appendix.

11. Turn to page 255 of your textbook. Do questions 1 to 4 of On Your Own.

12. Turn to page 257 of your textbook. Do questions 1 to 4 of Practise Your Skills.

13. Turn to page 259 of your textbook. Do question 1 of On Your Own.

If you need help with questions 11 to 13, look back at Lesson 3, where you found the probabilities for possible outcomes using various dice. If you feel you need more practice, do questions 14 and 15.

14. Turn to page 270 of your textbook. Do questions 3 and 5 of Skill Bank from This Unit.

15. Turn to pages 268 and 269 of your textbook. Do questions 2 and 3 of Problem Bank.

Check your answers on pages 96 to 98 in the Appendix.

If you need additional work to master the material in this module, work through Lesson 19: Probability on the Mathematics 6 Companion CD.

After completing this lesson, you can print out an assignment by clicking on the Activity button at the bottom of the page.

Ask your home instructor to print the solutions to the questions in the activity by clicking on the Parent Notes button at the bottom of the page.

Discuss your answers with your home instructor.

Just the Facts

Ask your home instructor to time you as you complete the following timed drill. Your goal is to complete all 25 questions in two minutes. At the end of two minutes, count how many questions you were able to complete. Then use the Answer Key in the Appendix to mark the drill, and record your score in the space provided. Before you move on, go back and complete any questions you did not finish.

Multiplication and Division Facts

$1 \times 2 =$	$18 \div 6 =$	$\begin{array}{r} 4 \\ \times 4 \\ \hline \end{array}$	$2\overline{)14}$	$8 \times 9 =$
$8 \div 1 =$	$\begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$	$8\overline{)48}$	$2 \times 6 =$	$56 \div 7 =$
$\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$	$6\overline{)24}$	$3 \times 4 =$	$8 \div 8 =$	$\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$
$3\overline{)21}$	$8 \times 0 =$	$20 \div 4 =$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$9\overline{)0}$
$5 \times 6 =$	$8 \div 4 =$	$\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$	$9\overline{)81}$	$6 \times 3 =$

Multiplication and Division Facts

Number completed in 2 minutes: ______
Number correct in 2 minutes: ______

Record your score on the Just the Facts Progress Chart.

Mental Math

One way to estimate division problems is to use basic facts to find **compatible** numbers. This tip will show you how to use this strategy along with **compensation**.

Example

Seventy-five hockey players agreed to share the \$463.50 it cost to rent a rink for a tournament. How much was each player's share?

Each player's share is \$463.50 ÷ 75.

To estimate the answer, you may consider doing any of the following:

- You may round 75 **down** to 70 and use division facts with 7 as a divisor.
  - If you round \$463.50 **down** to \$420, then \$420 ÷ 7 = \$60.
    So, \$420 ÷ 70 = \$6.
  - If you round \$463.50 **up** to \$490, then \$490 ÷ 7 = \$70.
    So, \$490 ÷ 70 = \$7.

- You may round 75 **up** to 80 and use division facts with 8 as a divisor.
  - If you round \$463.50 **down** to \$400, then \$400 ÷ 8 = \$50.
    So, \$400 ÷ 80 = \$5.
  - If you round \$463.50 **up** to \$480, then \$480 ÷ 8 = \$60.
    So, \$480 ÷ 80 = \$6.

Using a calculator, the exact answer is \$463.50 ÷ 75 = \$6.18.

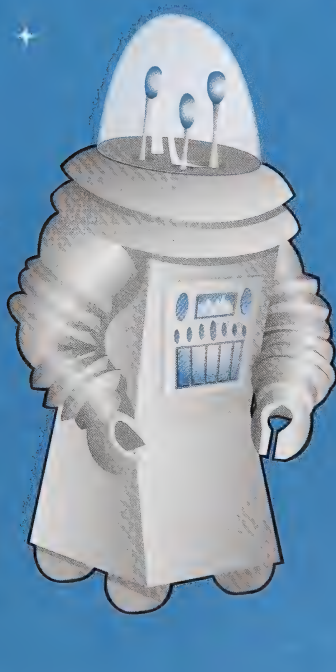
Notice that you get estimates **closest** to the exact answer when you round **both** the divisor and the dividend **up**, or when you round **both** the divisor and the dividend **down**.

Try to practise the preferred strategy whenever it is appropriate to use it.

Mathematics 6

Appendix

Glossary
Answer Key
Image Credits
Learning Aids



Glossary

certain event: an event that always occurs

experimental probability: a fraction between 0 and 1 determined by dividing the number of times a particular event is observed to occur by the total number of times the experiment is conducted

fair game: a game in which all the players have the same chance of winning

favourable outcomes: outcomes of an experiment in which a particular event under investigation occurs

impossible event: an event that never occurs

Platonic solid: a three-dimensional object with identical polygons as faces

possible outcomes: all the results of an experiment

prediction: a statement about what is expected to happen

probability: the study of chance

theoretical probability: a fraction between 0 and 1 determined by dividing the number of favourable outcomes by the total number of possible outcomes

Answer Key

Lesson 1: Making Predictions

Activity 1

1. **a.** can't happen, impossible, never
   b. absolutely, always, certain, definitely, for sure, must happen, will happen

2. **a.** improbable, unlikely
   b. should happen, usually, probable
   c. could happen, maybe, perhaps

3. **a.** It can't happen. It is impossible. It will never happen.

   b. It is either very unlikely and highly improbable, or it can't happen. It is utterly impossible. It will never happen.

4. **a.** The predictions of Lord Kelvin and Simon Newcombe were proven incorrect in 1903.

b. This tells us that tremendous advancements in the science of flight led to rapid and amazing accomplishments.

5. a. can't happen, impossible, never

   b. Orville's prediction was likely to be accepted as true because of the credibility he gained due to his and his brother's accomplishments five years earlier.

   c. Orville's prediction was proven incorrect 19 years later by Edward Rickenbacker.

6. a. Terms that describe what Edward Rickenbacker thought in 1924 and what Harry Bruno thought in 1943 are *absolutely*, *always*, *certain*, *definitely*, *for sure*, *must happen*, and *will happen*.

   b. Answers will vary. A sample answer is given.

   The chances of their predictions coming true are highly improbable. It is more likely that new technologies will result in different modes of transportation. Further advancements will certainly be sought, given the fact that gasoline is a non-renewable resource.

7. a. Answers will vary. A sample answer is given.

   Bickerton (1929) and Sir Richard van der Riet Wooley (1936) were proven incorrect in 1969.

   b. Sir H. Spencer Jones (September 1957) was proven incorrect one month later in October 1957.

8. Answers will vary. A sample answer is given.

   Lord Birkenhead's 1930 prediction actually happened much earlier than he thought it would. (A soft landing on Mars took place in 1971, about 60 years ahead of his prediction.)

9. a. In 1969, Dr. von Braun made three predictions of things to happen by 2000, but none of them actually took place.

   b. Scientists do have the ability to build a sizable moon operation, but the decision was to build and operate an international space station instead. Achieving a manned Mars landing could happen if the decision is made to do so. However, to fly men to the outer planets is highly improbable in the near future.

10. Answers will vary. A sample answer is given.

You might predict that you will have graduated from high school within 8 years. You are already in Grade 6 and, in 6 years time, you should be in Grade 12.

Activity 2

1.

Description of the Chance of Precipitation	Percent
Rain or snow is very likely.	80%
It's 50-50 on whether you get precipitation or not.	50%
If you go ahead with your outdoor plans, keep an eye on the weather.	70%
Consider the effect of precipitation on your plans for outdoor activities. The chance for no precipitation is only 3 in 10!	30%
Want to water your lawn? The odds are favourable that Mother Nature might give you some help.	60%
No precipitation is expected even though it may be cloudy.	0%
The occurrence of precipitation is a near certainty.	90%
Little likelihood of rain or snow; only 1 chance in 10.	10%
An umbrella is recommended. Consider alternate plans for outdoor activities that are susceptible to rain. It's not a good day to pave the driveway.	40%
Precipitation is a certainty.	100%
No precipitation is expected.	20%

2. Using percents that are multiples of 10 simplifies the task. It becomes easier to decide by ranking the descriptions.

3. No, it means that the chances are 40% of getting some rain for any part of the day.

4. a. business decisions: whether or not to hold an event outdoors, whether to provide mostly hot drinks or mostly cold drinks to spectators

   b. personal decisions: what clothes to wear, whether to take a bicycle or get a ride

5. It is *highly likely* that the model can be used to make accurate predictions for future Olympic Games.

6. A country with a large population would have more athletes from which to pick their best.

7. A country whose government spends a lot of money can train their athletes better and send more athletes to compete.

8. It may be easier to predict who will win a gold medal in high jumping because it is based completely on a measured amount, whereas events like diving are more subjective and are based on what a group of judges think.

9. Answers will vary. Sample answers are given.

   - In both cases, predictions are difficult to make because the outcomes are based on many factors.

   - When circumstances are similar, the same things may reoccur. Past performances of athletes give clues as to how well they are likely to perform at future Olympics. Similarly, meteorologists compare recent weather readings with past weather patterns.

Activity 3

1. About 6 000 000 Canadians saw the economy as their main concern.

$$20\% = \frac{20}{100} = \frac{1}{5} = \frac{6\ 000\ 000}{30\ 000\ 000}$$

Mental calculation: 30 000 000 ÷ 5 = 6 000 000

2. About 15 000 000 Canadians know the name of Canada's first prime minister.

$$\frac{1}{2} = \frac{15\ 000\ 000}{30\ 000\ 000}$$

Mental calculation: 30 000 000 ÷ 2 = 15 000 000

3. **a.** About 80 floral sales each day would be expected to be impulse purchases.

$$\frac{8}{10} = \frac{80}{100}$$

b. No, this does not mean the manager can expect that 80% of his total profits from sales would be from impulse purchases. The amount of profit from impulse purchases may be either greater or less than the profit from remaining purchases.

4. About 200 faulty light bulbs might be expected in the population of 10 000 light bulbs.

$$\frac{2}{100} = \frac{200}{10\,000}$$

5. If he sold 1000 pizzas every week, he would expect to sell about 230 each Friday.

$$9 + 10 + 12 + 13 + 23 + 21 + 12 = 100$$

He predicted that for every 100 pizzas sold, 23 would be on Friday.

$$\frac{23}{100} = \frac{230}{1000}$$

6. If he sold 500 pizzas every week, he would expect about 135 to be Hawaiian.

$$\frac{27}{100} = \frac{135}{500}$$

7. He might expect to sell about 14 Hawaiian pizzas on Tuesday. He expects to sell $10\% = \frac{1}{10}$ of his pizzas on Tuesday, so $\frac{1}{10}$ of $135 = 135 \div 10 = 13.5$.

8. Jack's prediction is correct because $\frac{19}{125} = \frac{152}{1000}$ ($8 \times 25 = 1000$, and $8 \times 19 = 152$).

Challenge Activity

The expected sum is 126.

- Each time the die is tossed, the probability for a particular outcome is $\frac{1}{6}$.

- Since Kylee tossed the die 36 times, each outcome (from 1 through 6) would be expected to occur 6 times.

  $$\frac{1}{6} = \frac{6}{36}$$

- $6 \times 1 = 6$
  $6 \times 2 = 12$
  $6 \times 3 = 18$
  $6 \times 4 = 24$
  $6 \times 5 = 30$
  $6 \times 6 = 36$

- $6 + 12 + 18 + 24 + 30 + 36 = 126$

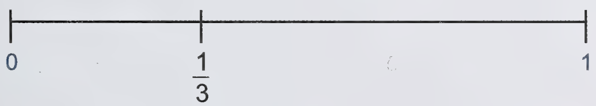
The expected sum of all the outcomes is 126.

Lesson 2: Outcomes and Chances

Activity 1

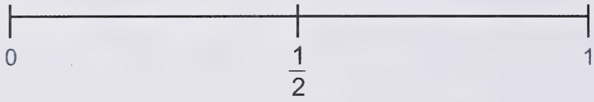
1. a. The chance of Bilinski being chosen as commander is 1 out of 3 because she is one of only three commanders available.

b. The probability of Bilinski being chosen as commander is $\frac{1}{3}$.

2. 

There are three possible commanders: Bilinski, Ho, and Evans. It states that each has an equal likelihood of being selected. Therefore, the probability of Ho being chosen is equal to the probability of Evans being chosen, which is the same as the probability of Bilinski being chosen.

3. a. The probability of a particular pilot being chosen is $\frac{1}{2}$ because one pilot is needed and there are two possible outcomes (Hardy and Goldberg).

b. 

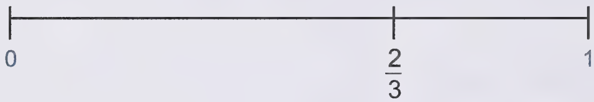
4. a. All the possible outcomes for picking pairs of mission specialists are

- Kasparov and Yano
- Kasparov and McHugh
- Yano and McHugh

b. There are three possible outcomes.

c. Each mission specialist's name appears in two of the outcomes.

d. The probability of a particular mission specialist being chosen is $\frac{2}{3}$ because each mission specialist occurs in two of the three possible outcomes.

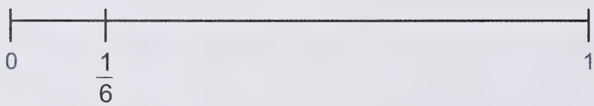
e. 

5. a. All the possible outcomes for a commander-pilot team are

- Bilinski and Hardy
- Bilinski and Goldberg
- Ho and Hardy
- Ho and Goldberg
- Evans and Hardy
- Evans and Goldberg

b. There are six possible outcomes.

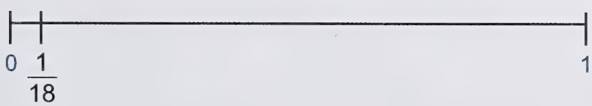
c. The probability of Evans and Hardy being chosen as the commander-pilot team is $\frac{1}{6}$ because they represent one out of six possible outcomes.

d. 

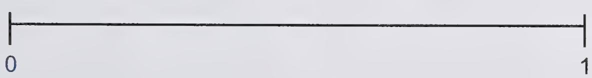
6.

	Commander	Pilot	Mission Specialists
Crew 1	Bilinski	Hardy	Kasparov and Yano
Crew 2	Bilinski	Hardy	Kasparov and McHugh
Crew 3	Bilinski	Hardy	Yano and McHugh
Crew 4	Bilinski	Goldberg	Kasparov and Yano
Crew 5	Bilinski	Goldberg	Kasparov and McHugh
Crew 6	Bilinski	Goldberg	Yano and McHugh
Crew 7	Ho	Hardy	Kasparov and Yano
Crew 8	Ho	Hardy	Kasparov and McHugh
Crew 9	Ho	Hardy	Yano and McHugh
Crew 10	Ho	Goldberg	Kasparov and Yano
Crew 11	Ho	Goldberg	Kasparov and McHugh
Crew 12	Ho	Goldberg	Yano and McHugh
Crew 13	Evans	Hardy	Kasparov and Yano
Crew 14	Evans	Hardy	Kasparov and McHugh
Crew 15	Evans	Hardy	Yano and McHugh
Crew 16	Evans	Goldberg	Kasparov and Yano
Crew 17	Evans	Goldberg	Kasparov and McHugh
Crew 18	Evans	Goldberg	Yano and McHugh

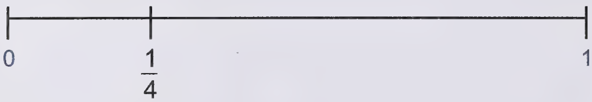
7. a. The probability of a particular crew being chosen is $\frac{1}{18}$ because each crew represents 1 out of 18 possible outcomes.

b. 

8. a. If Hardy is unavailable, the probability of Goldberg being chosen is 1 because that would leave Goldberg as the only possible pilot.

b. 

9. a. If Smith also became available, the probability of a particular commander being chosen is $\frac{1}{4}$ because one commander is needed and now there are four possible outcomes.

b. 

10. Kasparov is twice as likely to be chosen as Evans. The probability of Kasparov being chosen is $\frac{2}{3}$ and the probability of Evans being chosen is $\frac{1}{3}$. (Kasparov is on 12 out of 18 possible crews, and Evans is on 6 out of 18 possible crews.)

Activity 2

1. a. The light is green for 20 s in one cycle along Pelletier at Intersection 1 during daytime hours.

   b. One total cycle at Sale and Pelletier is 48 s during daytime hours.

   c. The probability of the light being green at noon is $\frac{20}{48}$.

   d. The probability in simplest terms is $\frac{20 \div 4}{48 \div 4} = \frac{5}{12}$.

2. a. $\frac{24 \div 24}{48 \div 24} = \frac{1}{2}$ b. $\frac{4 \div 4}{48 \div 4} = \frac{1}{12}$

3. a. $\frac{25 \div 5}{60 \div 5} = \frac{5}{12}$ b. $\frac{30 \div 30}{60 \div 30} = \frac{1}{2}$ c. $\frac{5 \div 5}{60 \div 5} = \frac{1}{12}$

4. Both traffic flows of Intersection 1 are programmed with the same timed cycle. Even though the daytime cycle is different from the nighttime cycle, the probability for Intersection 1 being green, red, or yellow is the same during daytime hours as it is during nighttime hours. The actual number of seconds for which the light is green, red, or yellow is different in the day than it is at night.

5. a. $\frac{20 \div 20}{40 \div 20} = \frac{1}{2}$ b. $\frac{16 \div 8}{40 \div 8} = \frac{2}{5}$ c. $\frac{4 \div 4}{40 \div 4} = \frac{1}{10}$

6. a. $\frac{10 \div 10}{40 \div 10} = \frac{1}{4}$ b. $\frac{26 \div 2}{40 \div 2} = \frac{13}{20}$ c. $\frac{4 \div 4}{40 \div 4} = \frac{1}{10}$

7. a. At Intersection 2, the light is not red along Quinn for the same length of time as it is either green or yellow along Hauk.

   20 + 4 does not equal 26, and 10 + 4 does not equal 16.

   The 2-s difference for the traffic flows means that for 2 s the lights are red for both flows of traffic. This causes a 2-s delay at Intersection 2.

   b. Answers will vary. A sample answer is given.

   The 2-s green-light delay makes Intersection 2 safer than Intersection 1. Compared to Intersection 1, there is a better chance that Intersection 2 will be clear before the other flow of traffic starts to enter. This reduces the chance of a collision in Intersection 2.

8. a. The greatest probability of the light being green is when travelling along Quinn through Intersection 2.

   b. The least probability of the light being green is when travelling along Hauk through Intersection 2.

9. Decisions about setting light cycle patterns at an intersection depend on which direction has the heaviest traffic flow. For example, based on the answers for question 8, it is highly likely that the traffic along Quinn Boulevard is much heavier than that along Hauk Drive. Also, the rates traffic flows may change with the times of day and/or the days of the week.

Activity 3

1. The six possible outcomes are ice cream, soft drink, hot dog, burger, fries, and muffin.

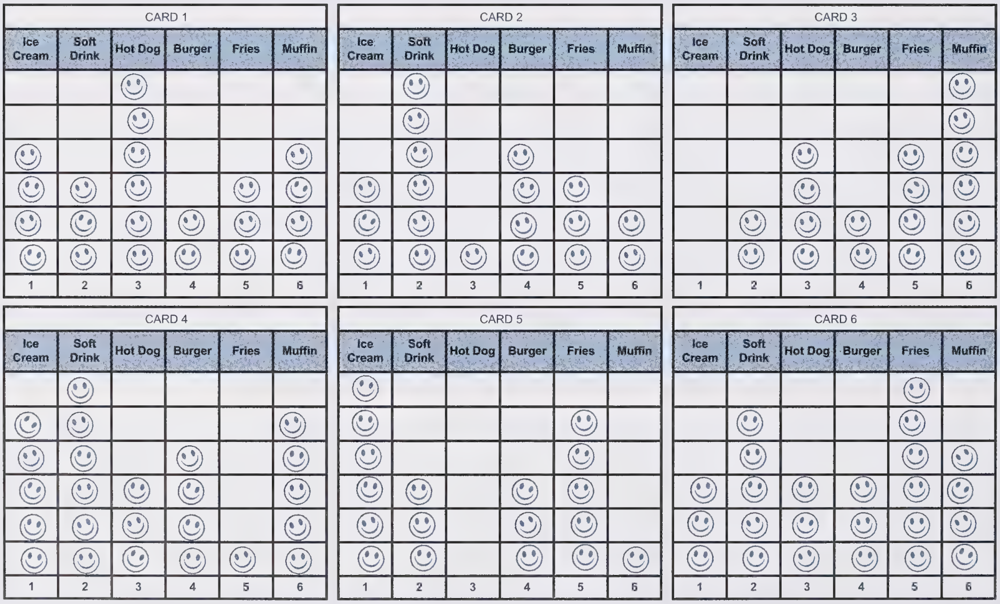
2. The outcomes are equally likely. Their theoretical probabilities are as follows:

- ice cream: $\frac{1}{6}$
- soft drink: $\frac{1}{6}$
- hot dog: $\frac{1}{6}$
- burger: $\frac{1}{6}$
- fries: $\frac{1}{6}$
- muffin: $\frac{1}{6}$

3. a. If you complete six game cards, you may predict that you will win one of each treat because each outcome is equally likely to happen.

   b. This prediction is not likely to happen. Even if you happen to win five different prizes after completing five game cards, it is more likely that one of those five prizes would repeat on the sixth card, rather than you winning a sixth different prize.

4. Answers will vary. Sample answers are given.



5. Answers will vary. The sample answers are based on the answers to question 4.

	Tallies					
Treat	**Card 1**	**Card 2**	**Card 3**	**Card 4**	**Card 5**	**Card 6**
Ice Cream	////	///		卌	卌	///
Soft Drink	///	卌	//	卌	///	卌
Hot Dog	卌	/	////	///		///
Burger	//	////	//	////	///	///
Fries	///	///	////	/	卌	卌
Muffin	////	//	卌	卌	/	////

6. Answers will vary. Sample answers are given.

   a. The experimental probabilities for the different possible outcomes are as follows:

   - ice cream: $\frac{1}{6}$
   - soft drink: $\frac{2}{6}$
   - hot dog: $\frac{1}{6}$
   - burger: $\frac{0}{6}$
   - fries: $\frac{1}{6}$
   - muffin: $\frac{1}{6}$

   b. The experimental results may be different from the theoretical probability because in only six games some treats are likely to repeat. This is what happened in the results shown above. A soft drink was won twice and a burger was not won.

7. Answers will vary. Sample answers are based on the answers to question 4 and 5.

	Frequencies						
Treat	**Card 1**	**Card 2**	**Card 3**	**Card 4**	**Card 5**	**Card 6**	**Total Frequency**
Ice Cream	4	3	0	5	6	3	21
Soft Drink	3	6	2	6	3	5	25
Hot Dog	6	1	4	3	0	3	17
Burger	2	4	2	4	3	3	18
Fries	3	3	4	1	5	6	22
Muffin	4	2	6	5	1	4	22
Total Frequency	22	19	18	24	18	24	125

8. Answers will vary. Sample answers are based on the answer to question 7.

   a. The bottom right cell of the table tells how many kids' meals were purchased to win six prizes.

   b. Compare the sum of the frequencies in the bottom row with the sum of the frequencies in the right column. If the sums are not equal, you know there is a mistake in the chart.

   c. A total of 125 kids' meals were purchased to win six treats.

   d. If 125 kids' meals were purchased, you would expect each treat to have been rolled about 21 times. Each outcome is expected $\frac{1}{6}$ of the time, and $125 \div 6$ is about 21.

e. The total frequency for most of the outcomes is close to the expected frequency of 21.

- Ice cream = 21 is exactly what was expected. Difference = 21 – 21 = 0
- Soft drink = 25 is 4 more than expected. Difference = 25 – 21 = 4
- Hot dog = 17 is 4 less than expected. Difference = 21 – 17 = 4
- Burger = 18 is 3 less than expected. Difference = 21 – 18 = 3
- Fries = 22 is 1 more than expected. Difference = 22 – 21 = 1
- Muffin = 22 is 1 more than expected. Difference = 22 – 21 = 1

9. a. The least number of kids' meals needed to be purchased to win a treat was 18.

b. The greatest number of kids' meals needed to be purchased to win a treat was 24.

c. The least possible number of kids' meals needed to be purchased to win a treat is six.

It is possible, but not likely, that you could toss the same number six times in a row.

d. The greatest possible number of kids' meals you would need to buy to win a treat is 31. If you stamped five squares in every column, that would make a total of 30 stamps. The next stamp, regardless of what was rolled, would win a treat.

e. You might predict that you would need to buy about 21 kids' meals. Since you needed 125 kids' meals to win 6 prizes, you might estimate that you would need $\frac{1}{6}$ of that number to win one prize. $125 \div 6$ is about 21.

Challenge Activity

1.

Number of Cookies the Same Flavour	List of Different Combinations	Number of Different Combinations
4	• 4 oatmeal • 4 chocolate chip • 4 raisin	3
3	• 3 oatmeal + 1 choc. chip • 3 oatmeal + 1 raisin • 3 choc. chip + 1 oatmeal • 3 choc. chip + 1 raisin • 3 raisin + 1 oatmeal • 3 raisin + 1 choc. chip	6
2	• 2 oatmeal + 2 choc. chip • 2 oatmeal + 2 raisin • 2 choc. chip + 2 raisin • 2 oatmeal + 1 choc. chip + 1 raisin • 2 choc. chip + 1 oatmeal + 1 raisin • 2 raisin + 1 oatmeal + 1 choc. chip	6
0		0

Total number of different combinations = 15

2. Greg could choose 15 different combinations of cookies.

Lesson 3: Games and Fairness

Activity 1

1. Answers will vary. Results for four sample games are given.

Game 1

Result	Tallies
White wins	///
Blue wins	////
Draw	///

Game 2

Result	Tallies
White wins	///
Blue wins	~~////~~ //
Draw	

Game 3

Result	Tallies
White wins	/
Blue wins	𝍸 /
Draw	///

Game 4

Result	Tallies
White wins	𝍸
Blue wins	𝍸
Draw	

2. There are three possible outcomes for each round: white wins, blue wins, and draw.

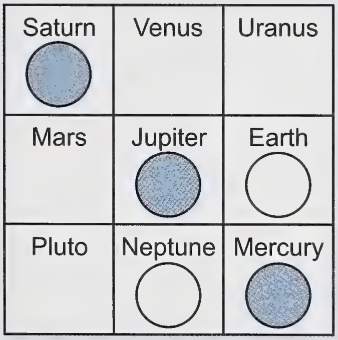
3. There are three possible outcomes for each game: white wins, blue wins, and tie.

4. A game is said to be fair when both (or all) players have an equally likely chance of winning.

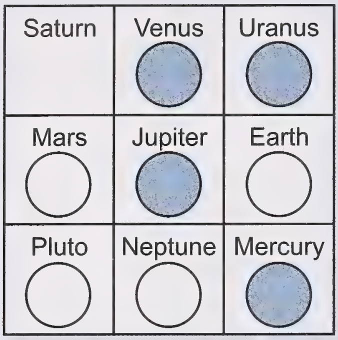
5. Theoretically speaking, the game is fair because both players have an equally likely chance of winning. They take turns going first and the cards are drawn by chance.

6. **a.** The least possible number of cards that must be drawn in all for a player to win a round is 5.

b. Answers will vary. A sample answer is shown.



7. Answers will vary. A sample answer is shown.



8. The double-win rule does not affect the fairness of the game. Both players have an equally likely chance of getting a double win.

9. You cannot become more skilful at this game if you play it often. There is no skill involved because no decisions of strategy are left for the players to make. The game is completely based on chance.

Activity 2

1.

Polyhedron	Shape of Faces	Consecutive Numbers Written on the Faces
Octahedron	equilateral triangles	1, 2, 3, 4, 5, 6, 7, 8
Tetrahedron	equilateral triangles	1, 2, 3, 4
Hexahedron	squares	1, 2, 3, 4, 5, 6
Icosahedron	equilateral triangles	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20
Dodecahedron	regular pentagons	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

2. Platonic solids are used for dice because when you toss one of them, it is equally likely to land on any of its faces. This means that all outcomes (numbers on its faces) are equally likely.

3. The hexahedron is likely the most popular shape used for a die because it is easy to make.

4.

		Probability of Favourable Outcome			
		Multiple of 5	Number Less Than 6	Multiple of 4	Prime Number
Die Used	Tetrahedron	0	$\frac{4}{4} = 1$	$\frac{1}{4}$	$\frac{2}{4} = \frac{1}{2}$
	Hexahedron	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{3}{6} = \frac{1}{2}$
	Octahedron	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{2}{8} = \frac{1}{4}$	$\frac{4}{8} = \frac{1}{2}$
	Dodecahedron	$\frac{2}{12} = \frac{1}{6}$	$\frac{5}{12}$	$\frac{3}{12} = \frac{1}{4}$	$\frac{5}{12}$
	Icosahedron	$\frac{4}{20} = \frac{1}{5}$	$\frac{5}{20} = \frac{1}{4}$	$\frac{5}{20} = \frac{1}{4}$	$\frac{8}{20} = \frac{4}{10} = \frac{2}{5}$

5. **a.** It is impossible to toss a multiple of 5 with a single tetrahedron.

   b. You have a 50% chance of tossing a prime number with the tetrahedron, the hexahedron, and the octahedron.

   c. You are very likely to toss a number less than 6 with the hexahedron. You are certain to do so with the tetrahedron.

6. **a.** It is possible for any particular player to win the game because each Platonic die has at least one multiple of 4 on it. Therefore, it is possible for any player to roll the most multiples of 4.

   b. This is not a fair game. The player using the hexahedron only has a probability of $\frac{1}{6}$ to toss a multiple of 4, but for all the other players the probability is $\frac{1}{4}$.

7. Yes, this is a fair game. The probability of tossing a prime number with a dodecahedral die is the same as the probability of tossing a number less than 6 with it. The probability of each $= \frac{5}{12}$. Therefore, each person would expect to get 5 points after 12 tosses.

8. Yes, this is a fair game to be played with an octahedral die. You are twice as likely to toss a multiple of 4 (2 chances out of 8) than you are to toss a multiple of 5 (1 chance out of 8), but you get twice as many points for tossing a multiple of 5 (2 points) as you do for tossing a multiple of 4 (1 point). So, after 8 tosses, each is expected to get 2 points.

9. You are least likely to roll a prime number with an icosahedral die. The probability of tossing a prime number with each is shown.

- icosahedron = $\frac{2}{5}$ and $\frac{2 \times 12}{5 \times 12} = \frac{24}{60}$
- dodecahedron = $\frac{5}{12}$ and $\frac{5 \times 5}{12 \times 5} = \frac{25}{60}$
- any of the other Platonic dice = $\frac{1 \times 15}{2 \times 15} = \frac{30}{60}$

Activity 3

1. The faces of the cube must be numbered as follows: one 3, three 4s, one 5, and one 6.

2. a. Answers will vary. Sample results of tossing the cube 30 times are given.

Outcome	Tallies	Frequency
3	//	2
4	~~////~~ ~~////~~ ~~////~~ ///	18
5	~~////~~ //	7
6	///	3

b.

Outcome	Expected Results for 30 tosses (theoretical)
3	$\frac{1 \times 5}{6 \times 5} = \frac{5}{30}$ (3 will be rolled 5 times)
4	$\frac{3 \times 5}{6 \times 5} = \frac{15}{30}$ (4 will be rolled 15 times)
5	$\frac{1 \times 5}{6 \times 5} = \frac{5}{30}$ (5 will be rolled 5 times)
6	$\frac{1 \times 5}{6 \times 5} = \frac{5}{30}$ (6 will be rolled 5 times)

c. Answers will vary. A sample answer is given.

The experimental outcomes were similar to the expected outcomes in that the outcome 4 occurred most often.

3. The faces of the tetrahedron must be coloured as follows: 1 blue, 1 yellow, and 2 red.

4. a.

Outcome	Theoretical Probability
Blue	$\frac{1}{4}$
Yellow	$\frac{1}{4}$
Red	$\frac{2}{4} = \frac{1}{2}$

b.

Outcome	Expected Results for 32 Tosses
Blue	$\frac{1 \times 8}{4 \times 8} = \frac{8}{32}$
Yellow	$\frac{1 \times 8}{4 \times 8} = \frac{8}{32}$
Red	$\frac{1 \times 16}{2 \times 16} = \frac{16}{32}$

c. Answers will vary. Sample results of tossing the tetrahedron 32 times are given.

Outcome	Tallies	Frequency
Blue	卌	5
Yellow	卌 ////	9
Red	卌 卌 卌 ///	18

d. Answers will vary. A sample answer is given.

The experimental results were similar to the theoretical (expected) results. The outcome red occurred most often, only two more times than expected.

5. The faces of the octahedron must be marked as follows: 2 with an **X** and 6 with an **O**.

6. a.

Outcome	Probability
X	$\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$
O	$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

b.

Outcome	Equivalent Fraction	Expected Frequency After 40 Tosses
X	$\frac{2 \times 5}{8 \times 5} = \frac{10}{40}$	You would expect 10 **X**s.
O	$\frac{3 \times 10}{4 \times 10} = \frac{30}{40}$	You would expect 30 **O**s.

c. Answers will vary. Sample results of tossing the octahedron 40 times are given.

Outcome	Tallies	Frequency
X	卌 |||	8
O	卌 卌 卌 卌 卌 卌 ||	32

d. Answers will vary. A sample answer is given.

The experimental results were similar to what was expected (theoretical results).

7. a. The possible outcomes for tossing the pyramid are 1, 2, 3, 4, and 5. They are not all equally likely. The pyramid is more likely to land on its square base than on any one of its triangular faces because the base has a greater area than any one of the four triangular faces. Therefore, the number written on the square base is more likely to occur than any other number. Also, because the areas of the four triangular faces are equal to each other, the numbers written on those faces are equally likely to occur.

b. Answers will vary. A sample answer is given.

Outcome	The Guess for the Frequency After 20 Tosses
1 (the square base)	8
2 (a triangular face)	3
3 (a triangular face)	3
4 (a triangular face)	3
5 (a triangular face)	3

c. Answers will vary. Sample experimental results of tossing the die 20 times are given:

Outcome	Tallies	Frequency
1 (the square base)	~~||||~~ |||	8
2 (a triangular face)	||	2
3 (a triangular face)	~~||||~~	5
4 (a triangular face)	|||	3
5 (a triangular face)	||||	4

d. Answers will vary. A sample answer is given.

The experimental outcomes were similar to the expected outcomes, particularly in that the die landed on the square base more than any other face.

8. a. For the outcomes of the die to be equally likely, it is best to have all of its faces the same size and shape.

b. If you only want to use one die, then use a polyhedron that has the same number of faces as the total number of different outcomes wanted. Cubes are most often used because they are the simplest and the least expensive to make. (By using more than one die, different numbers of outcomes with different probabilities are possible.)

Challenge Activity

1. Cubes A and D are made from the net that is shown.

2. a. Cube A: The number 5 will be on its bottom face.
Cube D: The number 6 will be on its bottom face.

b. You can tell by looking at the net because the top and bottom numbers cannot be on adjacent squares. (They have to be separated by a square.)

Cube A: Square 1 and Square 5 are separated by Square 3.
Cube D: Square 3 and Square 6 are separated by Square 5.

3. Cube A: The number 4 will be on its hidden face to the left.
   Cube D: The number 1 will be on its hidden face to the left.

4. Cube A: The number 6 will be on its hidden face to the right.
   Cube D: The number 2 will be on its hidden face to the right.

Keystrokes

1. The ten thousands and thousands digits show that 36 tosses have been made and entered. The hundreds, tens, and ones digits tell the final total.

2. The least possible final display is 36072. You would get a final total of 72 if you tossed two 1s each time. This is very unlikely.

3. The greatest possible final display is 36432. You would get a final total of 432 if you tossed two 6s each time. This is very unlikely.

4. This is a fair game because the outcomes are equally likely for all players.

5. You cannot become more skilful at this game if you play it often. The outcomes depend entirely on chance.

6. This is a fair game because the outcomes on each toss depend on chance.

7. Some strategies for choosing the number of dice are as follows:

   - At the beginning, using the most dice possible each time allows you to subtract large numbers so that your points decrease quickly.

   - If your display is a three-digit number with a small number in the hundreds place, you may lose your turn if you toss three dice. If you use two dice, you are certain to be able to subtract points.

8. You can become more skilful at making choices about how many dice to toss if you play this game often.

Review

1. Textbook, page 250, Starting Out

- Umpire: This game is moving so slowly, I probably won't be home in time for dinner.

- Fan dressed in blue: It's impossible for them to win this game.

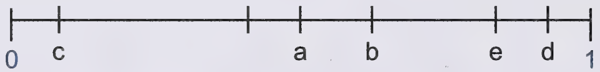
- Batter: If he throws his curve ball, it is very probable that I'll hit it.

- Fan wearing orange hat and green shirt: I wonder who will win this game? I wonder what the final score will be?

- Pitcher: She's a fabulous batter—it's not possible for me to strike her out.

- Coach: If we win and the Suns lose, the next team we play will probably be either the Bulldogs or the Grizzlies.

2. The probabilities are shown on the following number line.



3. Textbook, page 255, Practise Your Skills, questions 1 to 3

1. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16}$

2. $\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{12}{40} = \frac{15}{50} = \frac{18}{60}$

3. $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28} = \frac{24}{32} = \frac{27}{36}$

4. The statements are written using different words as follows:

- My soccer team *might* win tomorrow's game.
- There *probably* will be a large crowd of spectators at tomorrow's soccer game.
- The visiting team says it's *impossible* for us to beat them.

5. **Textbook, page 262, Practise Your Skills, questions 1 to 8**

1. $\frac{1}{6} = \frac{6}{36}$
2. $\frac{1}{8} = \frac{6}{48}$
3. $\frac{1}{3} = \frac{20}{60}$
4. $\frac{1}{5} = \frac{10}{50}$
5. $\frac{3}{4} = \frac{45}{60}$
6. $\frac{2}{3} = \frac{32}{48}$
7. $\frac{3}{8} = \frac{18}{48}$
8. $\frac{2}{5} = \frac{16}{40}$

6. **Textbook, page 252, Starting Out**

a. For each line-up, the following positions are chosen: two on defence, one right winger, one left winger, and one centre, with a mix of male and female players on each team.

b. Answers will vary. Sample answers are given.

Position	Different Line-up 1	Different Line-up 2
Offence	Alison Stern C Mark Goode RW Timothy Brown LW	Kenny Chan C Hsiao Chang RW Jennie Franklin LW
Defence	Faraz Ahned Winston White	Sam Rajnathan Cherie Roberts

c. The schedule of fall games shows all possible pairs of teams.

d. Another 15 games must be scheduled so that every team hosts every other team once.

7. **Textbook, page 262, On Your Own, question 2**

2. You would expect to spin red 10 out of 40 times ($\frac{1}{4}$ of the time). There are 8 equal sectors on the spinner and 2 are red: $\frac{2}{8} = \frac{1}{4}$, and $40 \div 4 = 10$.

8. **Textbook, page 270, Skill Bank from This unit, questions 2 and 4**

2. $\frac{36}{60} = \frac{3}{5}$ of the spins landed on red and $\frac{24}{60} = \frac{2}{5}$ of the spins landed on blue.

4. a. $\frac{2}{6} = \frac{1}{3}$ b. $\frac{4}{12} = \frac{1}{3}$ c. $\frac{2}{8} = \frac{1}{4}$

9. Textbook, page 264

You can combine a shirt with a pair of pants in 12 different ways.

Shirt	Pants
brown	brown
brown	black
brown	blue

Shirt	Pants
white	brown
white	black
white	blue

Shirt	Pants
beige	brown
beige	black
beige	blue

Shirt	Pants
blue	brown
blue	black
blue	blue

10. Textbook, page 291, Skill Bank Looking Back

The probability you will choose vegetable soup and an egg salad sandwich is $\frac{1}{6}$.
You can combine a soup and a sandwich in six ways.

Soup	Sandwich
chicken	roast beef
chicken	tuna
chicken	egg salad

Soup	Sandwich
vegetable	roast beef
vegetable	tuna
vegetable	egg salad

11. Textbook, page 255, On Your Own, questions 1 to 4

Answers will vary. Sample answers are given.

1. It will probably land on heads about 25 times.

2.

Outcome	Tallies	Frequency
Heads	卌 卌 卌 卌 |||	23
Tails	卌 卌 卌 卌 卌 ||	27

3. $\frac{23}{50}$ of the tosses landed heads and $\frac{27}{50}$ of the tosses landed tails.

4. The prediction of half the tosses landing heads was close.

12. Textbook, page 257, Practise Your Skills, questions 1 to 4

1. Probability of red is $\frac{1}{4}$.

2. Probability of blue is $\frac{6}{6} = 1$ (It is certain).

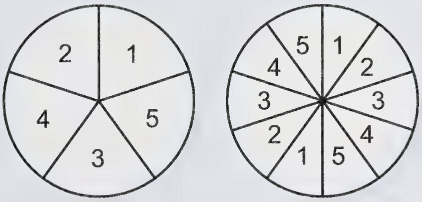
3. Probability of green is $\frac{6}{8} = \frac{3}{4}$ and the probability of white is $\frac{2}{8} = \frac{1}{4}$.

4. Probability of red is $\frac{3}{12} = \frac{1}{4}$, probability of blue is $\frac{8}{12} = \frac{2}{3}$, and probability of yellow is $\frac{1}{12}$.

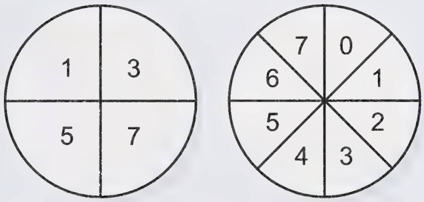
13. Textbook, page 259, On Your Own, question 1

1. Answers will vary. Sample answers are given.

   a. Probability of 2 is $\frac{1}{5}$.

   b. Probability of less than 6 is $\frac{3}{4}$.

14. Textbook, page 270, Skill Bank from This unit, questions 3 and 5

3. **a.** For the regular tetrahedron, the probability of A is $\frac{3}{4}$.
   b. For the cube, the probability of A is $\frac{3}{6} = \frac{1}{2}$.
   c. For the regular octahedron, the probability of A is $\frac{3}{8}$.

5. Answers will vary. Sample answers are given.

   a. Possible numbers: 1, 1, 1, 2, 2, 2 or 1, 1, 1, 4, 5, 6
   b. Possible numbers: 3, 3, 3, 3, 4, 4 or 1, 2, 3, 4, 5, 7

15. Textbook, pages 268 and 269, Problem Bank, questions 2 and 3

2. The probabilities of the outcomes are as follows:

a. A spade is $\frac{13}{52} = \frac{1}{4}$.

b. A red card is $\frac{26}{52} = \frac{1}{2}$.

c. A queen is $\frac{4}{52} = \frac{1}{13}$.

d. A 4 is $\frac{4}{52} = \frac{1}{13}$.

e. A face card is $\frac{12}{52} = \frac{3}{13}$.

f. A 15 is $\frac{0}{52} = 0$.

g. A number from 2 to 10 (including 2 and 10) is $\frac{36}{52} = \frac{9}{13}$.

3. Answers will vary. Sample answers are given.

a. One face on the white cube is 6. Each of the other five faces can be 1, 2, 3, 4, or 5. (e.g., 1, 2, 3, 4, 5, 6 or 1, 1, 4, 4, 5, 6)

Five faces on the yellow cube are even. The other face must be odd. (e.g., 2, 2, 4, 4, 6, 1 or 3, 4, 4, 4, 6, 6)

b. Three faces on the white cube are 1. Each of the other three faces can be 2, 3, 4, 5, or 6. (e.g., 1, 1, 1, 4, 4, 5 or 1, 1, 1, 2, 3, 5)

Four faces on the yellow cube must have either a 1, 2, or 3 written on them. The other two faces must have either a 4, 5 or 6 written on them. (e.g., 1, 1, 1, 1, 5, 5 or 1, 2, 3, 3, 5, 6)

Just the Facts

2	3	16	7	72
8	12	6	12	8
0	4	12	1	9
7	0	5	56	0
30	2	0	9	18

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CARD 1					
Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

CARD 2					
Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

CARD 3

Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

CARD 4

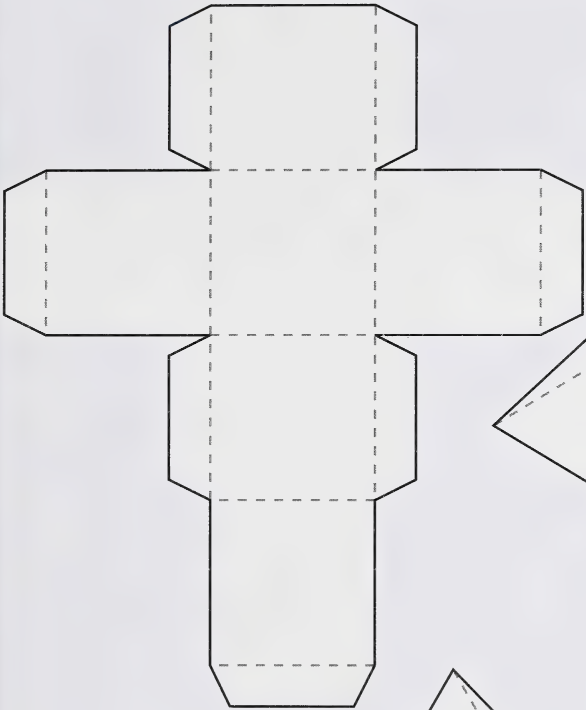
Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

CARD 5					
Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

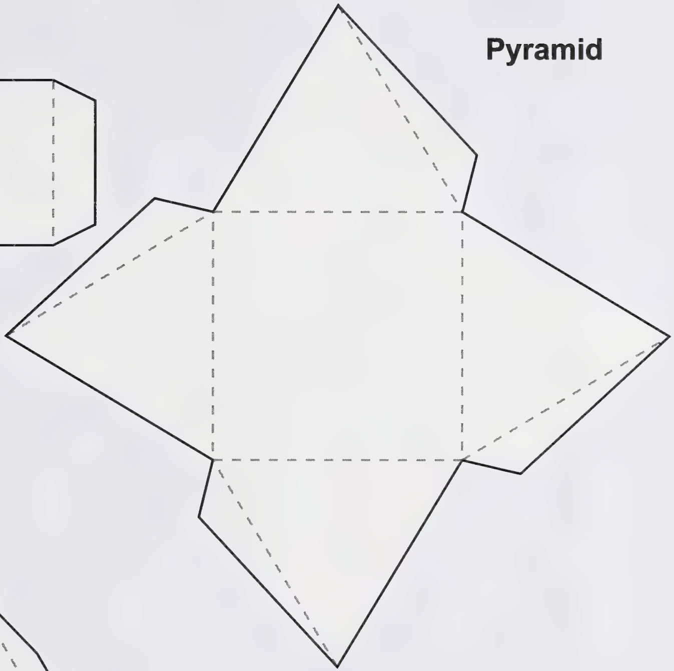
CARD 6					
Ice Cream	Soft Drink	Hot Dog	Burger	Fries	Muffin
1	2	3	4	5	6

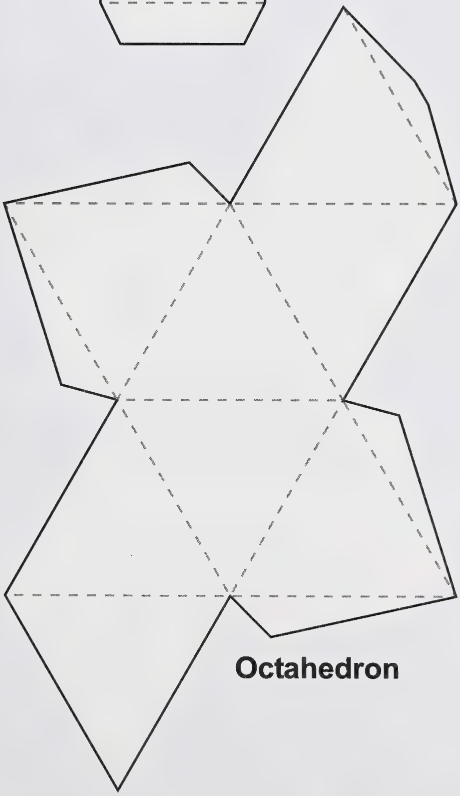
Nets

Cube

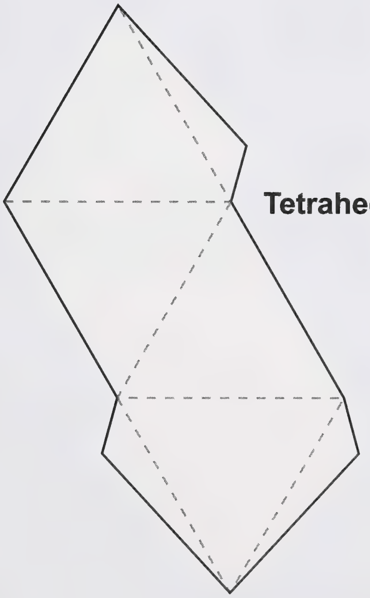


Pyramid



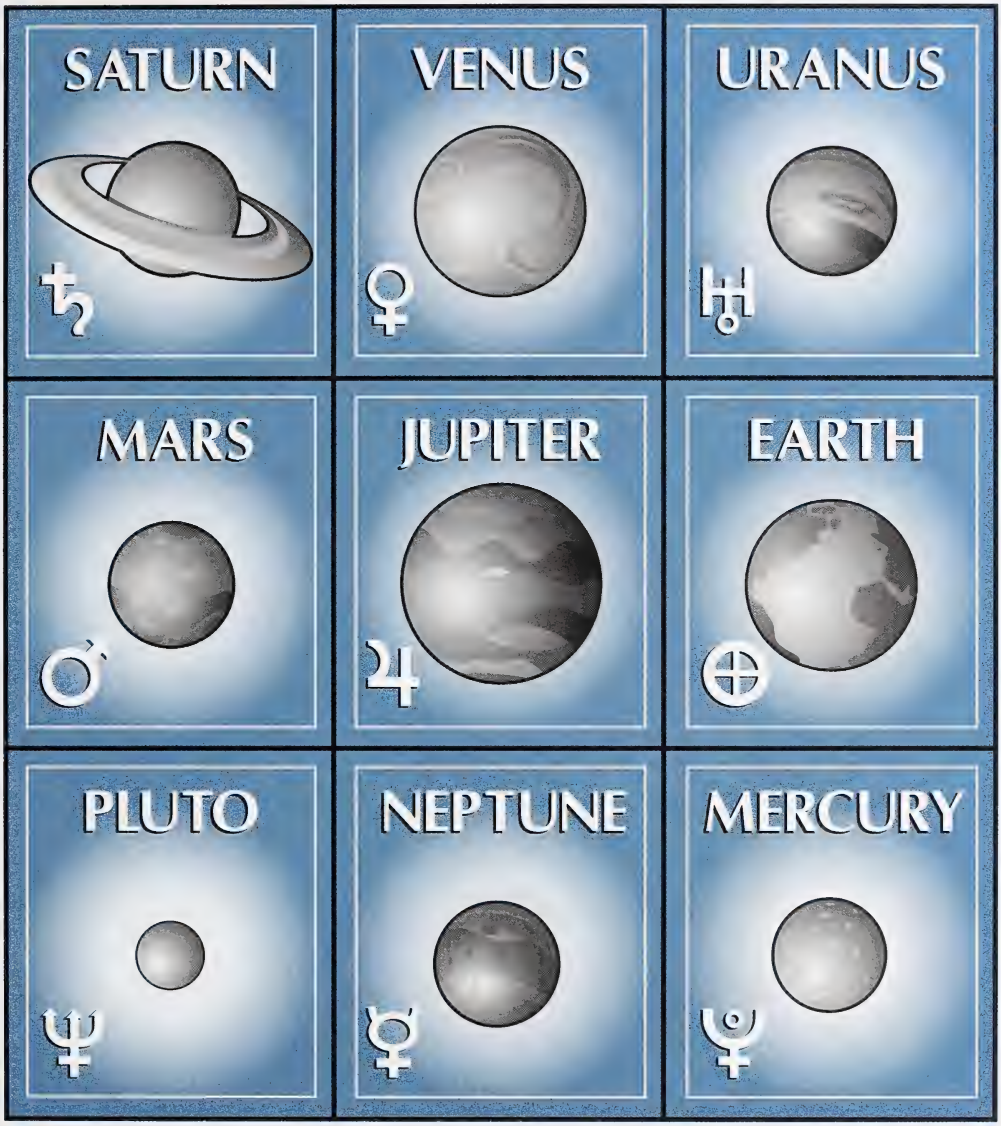


Octahedron



Tetrahedron

Planets Aligned Game Board



Planets Aligned Game Pieces